VISIT…

SECTION 18

CAMS

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Generating Cam Curves

It usually doesn't pay to design a complex cam curve if it can't be easily machined–so check these mechanisms before starting your cam design.

Preben W. Jensen

IF you have to machine a cam curve into the metal blank without using a master cam, how accurate can you expect it to be? That depends primarily on how precisely the mechanism you use can feed the cutter into the cam blank. The mechanisms described here have been carefully selected for their practicability. They can be employed directly to machine the cams, or to make master cams for producing others.

The cam curves are those frequently employed in automatic-feed mechanisms and screw machines. They are the circular, constant-velocity, simple-harmonic, cycloidal, modified cycloidal, and circular-arc cam curve, presented in that order.

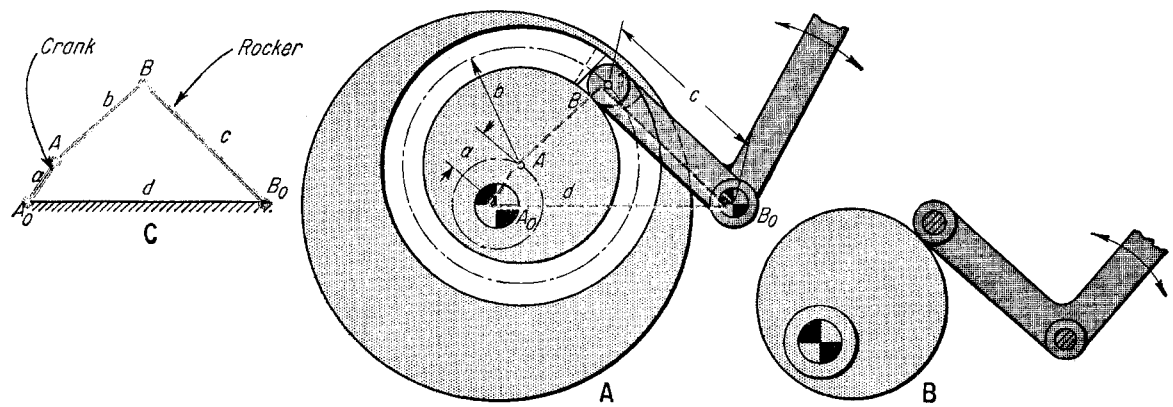
Circular cams

This is popular among machinists because of the ease in cutting the groove. The cam (Fig 1A) has a circular groove whose center, A, is displaced a distance a from the cam-plate center, A_0, or it may simply be a plate cam with a spring-loaded follower (Fig 1B).

Interestingly, with this cam you can easily duplicate the motion of a four-bar linkage (Fig 1C). Rocker BB_0 in Fig 1C, therefore, is equivalent to the motion of the swinging follower in Fig 1A.

The cam is machined by mounting the plate eccentrically on a lathe. The circular groove thus can be cut to close tolerances with an excellent surface finish.

If the cam is to operate at low speeds you can replace the roller with an arc-formed slide. This permits the transmission of high forces. The optimum design of such "power cams" usually requires time-consuming computations, but charts were published re-



1. Circular cam groove is easily machined on turret lathe by mounting the plate eccentrically onto the truck. Plate cam in **(B)** with spring load follower produces same output motion. Many designers are unaware that this type of cam has same output motion as four-bar linkage **(C)** with the indicated equivalent link lengths. Hence it's the easiest curve to pick when substituting a cam for an existing linkage.

cently (see Editor's Note at end of article) which simplify this aspect of design.

The disadvantage (or sometimes, the advantage) of the circular-arc cam is that, when traveling from one given point, its follower reaches higher speed accelerations than with other equivalent cam curves.

Constant-velocity cams

A constant-velocity cam profile can be generated by rotating the cam plate and feeding the cutter linearly, both with uniform velocity, along the path the translating roller follower will travel later (Fig 2A). In the case of a swinging follower, the tracer (cutter) point is placed on an arm equal to the length of the actual swinging roller follower, and the arm is rotated with uniform velocity (Fig 2B).

Simple-harmonic cams

The cam is generated by rotating it with uniform velocity and moving the cutter with a scotch yoke geared to the rotary motion of the cam. Fig 3A shows the principle for a radial translating follower; the same principle is, of course, applicable for offset translating and swinging roller follower. The gear ratios and length of the crank working in the scotch yoke control the pressure angles (the angles for the rise or return strokes).

For barrel cams with harmonic motion the jig in Fig 3B can easily be set up to do the machining. Here, the barrel cam is shifted axially by means of the rotating, weight-loaded (or spring-loaded) truncated cylinder.

The scotch-yoke inversion linkage (Fig 3C) replaces the gearing called for in Fig 3A. It will cut an approximate simple-harmonic motion curve when the cam has a swinging roller follower, and an exact curve when the cam has a radial or offset translating roller follower. The slotted member is fixed to the machine frame *1*. Crank *2* is driven around the center *0*. This causes link *4* to oscillate back and forward in simple harmonic motion. The sliding piece *5* carries the cam to be cut, and the cam is rotated around the center of *5* with uniform velocity. The length of arm *6* is made equal to the length of the swinging roller follower of the actual cam mechanism and the device adjusted so that the extreme positions of the center of *5* lie on the center line of *4*.

The cutter is placed in a stationary spot somewhere along the centerline of member *4*. In case a radial or offset translating roller follower is used, the sliding piece *5* is fastened to *4*.

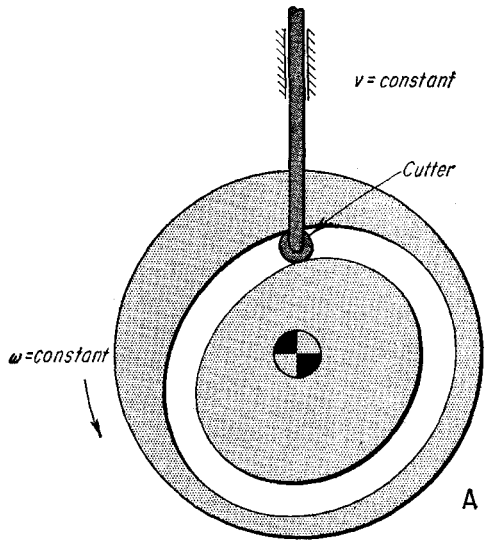
The deviation from simple harmonic motion when the cam has a swinging follower causes an increase in acceleration ranging from 0 to 18% (Fig 3D), which depends on the total angle of oscillation of the follower. Note that for a typical total oscillating angle of 45 deg, the increase in acceleration is about 5%.

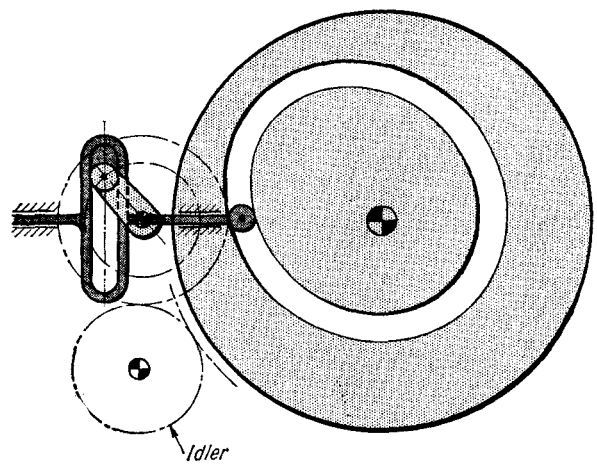
Cycloidal motion

This curve is perhaps the most desirable from a designer's viewpoint because of its excellent acceleration characteristic. Luckily, this curve is comparatively easy to generate. Before selecting the mechanism it is worthwhile looking at the underlying theory of the cycloids because it is possible to generate not only cycloidal motion but a whole family of similar curves.

The cycloids are based on an offset sinusoidal wave (Fig 4). Because the radii of curvatures in points *C*, *V*, and *D* are infinite (the curve is "flat" at these points), if this curve was a cam groove and moved in the direction of line *CVD*, a translating roller follower, actuated by this cam, would have zero acceleration at points *C*, *V*, and *D* no matter in what direction the follower is pointed.

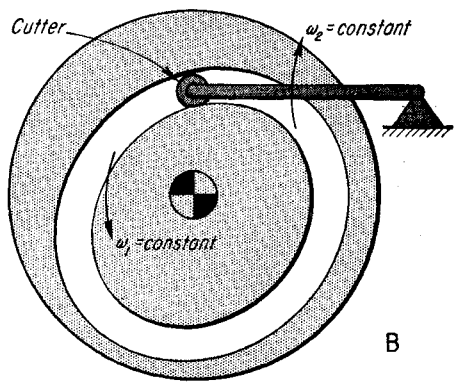
Now, if the cam is moved in the direction of *CE* and the direction of motion of the translating follower is lined perpendicular to *CE*, the acceleration of the follower in points *C*, *V*, and *D* would still be zero.





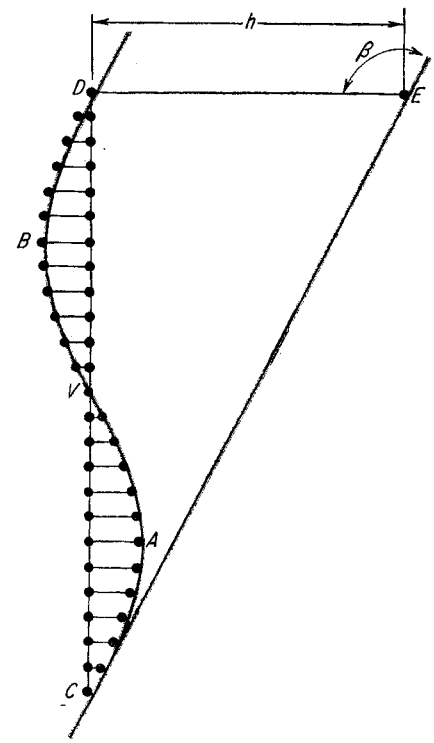
3. For producing simple harmonic curves: (A) Scotch yoke device feeds cutter while gearing arrangement rotates cam; **(B)** truncated-cylinder slider for

2. Constant-velocity cam is machined by feeding the cutter and rotating the cam at constant velocity. Cutter is fed linearly **(A)** or circularly **(B)**, depending on type of follower.

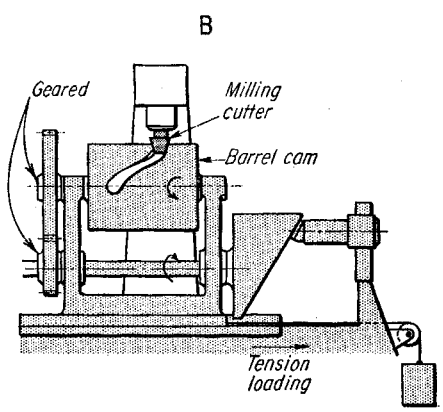


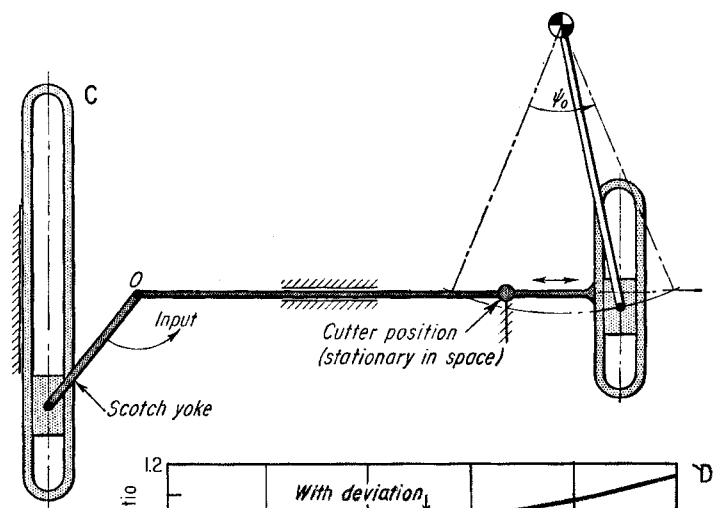
This has now become the basic cycloidal curve, and it can be considered as a sinusoidal curve of a certain amplitude (with the amplitude measured perpendicular to the straight line) superimposed on a straight (constant-velocity) line.

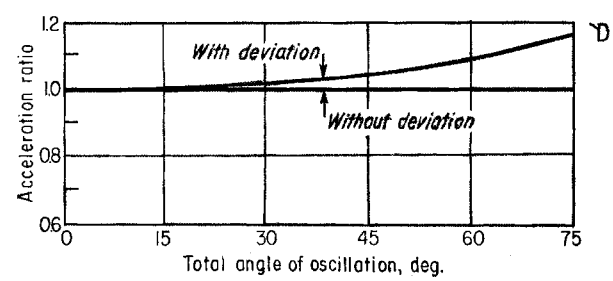
The cycloidal is considered the best *standard* cam contour because of its low dynamic loads and low



4. Layout of a cycloidal curve.







cylindrical cam; **(C)** scotch-yoke inversion linkage for avoiding gearing; **(D)** increase in acceleration when translating follower is replaced by swinging follower.

shock and vibration characteristics. One reason for these outstanding attributes is that it avoids any sudden change in acceleration during the cam cycle. But improved performances are obtainable with certain modified cycloidals.

Modified cycloids

To get a modified cycloid, you need only change the direction and magnitude of the amplitude, while keeping the radius of curvature infinite at points *C*, *V*, and *D*.

Comparisons are made in Fig 5 of some of the modified curves used in industry. The true cycloidal is shown in the cam diagram of *A*. Note that the sine amplitudes to be added to the constant-velocity line are perpendicular to the base. In the Alt modification shown in *B* (after Hermann Alt, German kinematician, who first analyzed it), the sine amplitudes are perpendicular to the constant-velocity line. This results in improved (lower) velocity characteristics (see *D*), but higher acceleration magnitudes (see *E*).

The Wildt modified cycloidal (after Paul Wildt) is constructed by selecting a point *w* which is 0.57 the distance *T/2*, and then drawing line *wp* through *yp* which is midway along *OP*. The base of the sine curve is then constructed perpendicular to *yw*. This modification results in a maximum acceleration of 5.88 h/T^2, whereas the standard cycloidal curve has a maximum acceleration of 6.28 h/T^2. This is a 6.8% reduction in acceleration.

(It's quite a trick to construct a cycloidal curve to go through a particular point *P*—where *P* may be anywhere within the limits of the box in *C*—and with a specific slope at *P*. There is a growing demand for this type of modification, and a new, simple, graphic technique developed for meeting such requirements will be shown in the next issue.)

Generating the modified cycloidals

One of the few devices capable of generating the family of modified cycloidals consists of a double carriage and rack arrangement (Fig 6A).

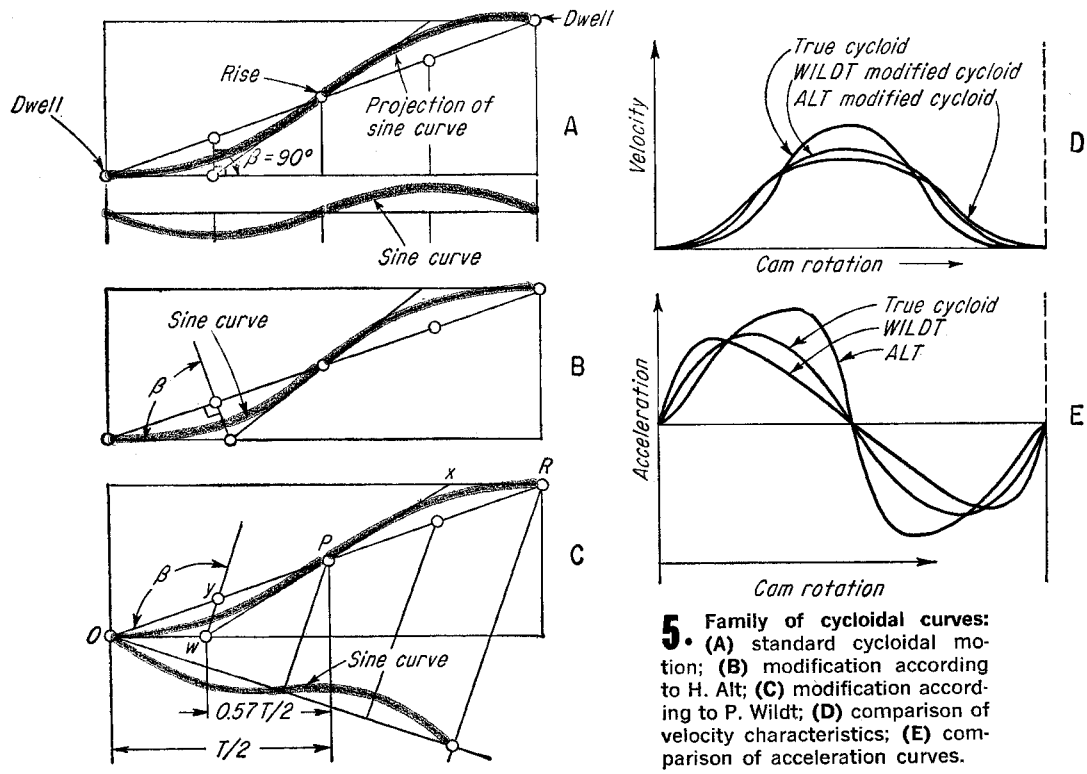
The cam blank can pivot around the spindle, which in turn is on the movable carriage *I*. The cutter center is stationary. If the carriage is now driven at constant speed by the lead screw, in the direction of the arrow, the steel bands 1 and 2 will also cause the cam blank to rotate. This rotation-and-translation motion to the cam will cause a spiral type of groove.

For the modified cycloidals, a second motion must be imposed on the cam to compensate for the deviations from the true cycloidal. This is done by a second steel band arrangement. As carriage *I* moves, the bands 3 and 4 cause the eccentric to rotate. Because of the stationary frame, the slide surrounding the eccentric is actuated horizontally. This slide is part of carriage *II*, with the result that a sinusoidal motion is imposed on to the cam.

Carriage *I* can be set at various angles β to match angle β in Fig 5B and C. The mechanism can also be modified to cut cams with swinging followers.

Circular-arc cams

Although in recent years it has become the custom to turn to the cycloidal and other similar curves even when speeds are low, there are many purposes for which



5. **Family of cycloidal curves: (A)** standard cycloidal motion; **(B)** modification according to H. Alt; **(C)** modification according to P. Wildt; **(D)** comparison of velocity characteristics; **(E)** comparison of acceleration curves.

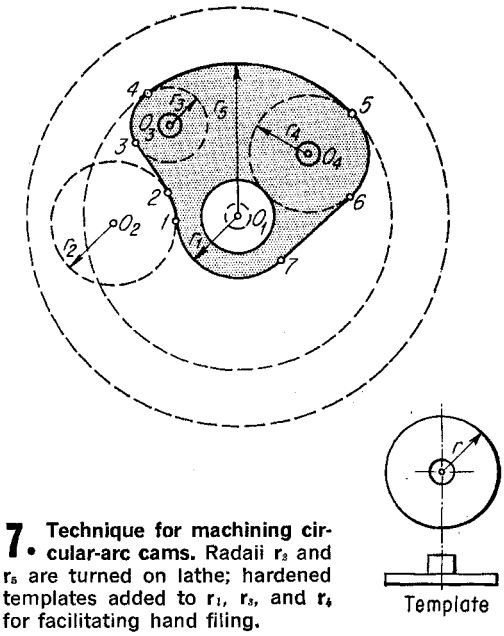
circular-arc cams suffice. Such cams are composed of circular arcs, or circular arcs and straight lines. For comparatively small cams the cutting technique illustrated in Fig 7 produces good accuracy.

Assume that the contour is composed of circular arc *1-2* with center at O_2, arc *3-4* with center at O_3, arc *4-5* with center at O_1, arc *5-6* with center at O_4, arc *7-1* with center at O_1, and the straight lines *2-3* and *6-7*. The method involves a combination of drilling, lathe turning, and template filing.

First, small holes about 0.1 in diameter are drilled at O_1, O_3, and O_4, then a hole is drilled with the center at O_2 and radius of r_2. Next the cam is fixed in a turret lathe with the center of rotation at O_1, and the steel plate is cut until it has a diameter of $2r_5$. This takes care of the larger convex radius. The straight lines *6-7* and *2-3* are now milled on a milling machine.

Finally, for the smaller convex arcs, hardened pieces are turned with radii r_1, r_3, and r_4. One such piece is shown in Fig 7B. The templates have hubs which fit into the drilled holes at O_1, O_3, and O_4. Now the arc *7-1*, *3-4*, and *5-6* are filed, using the hardened templates as a guide. Final operation is to drill the enlarged hole at O_1 to a size that a hub can be fastened to the cam.

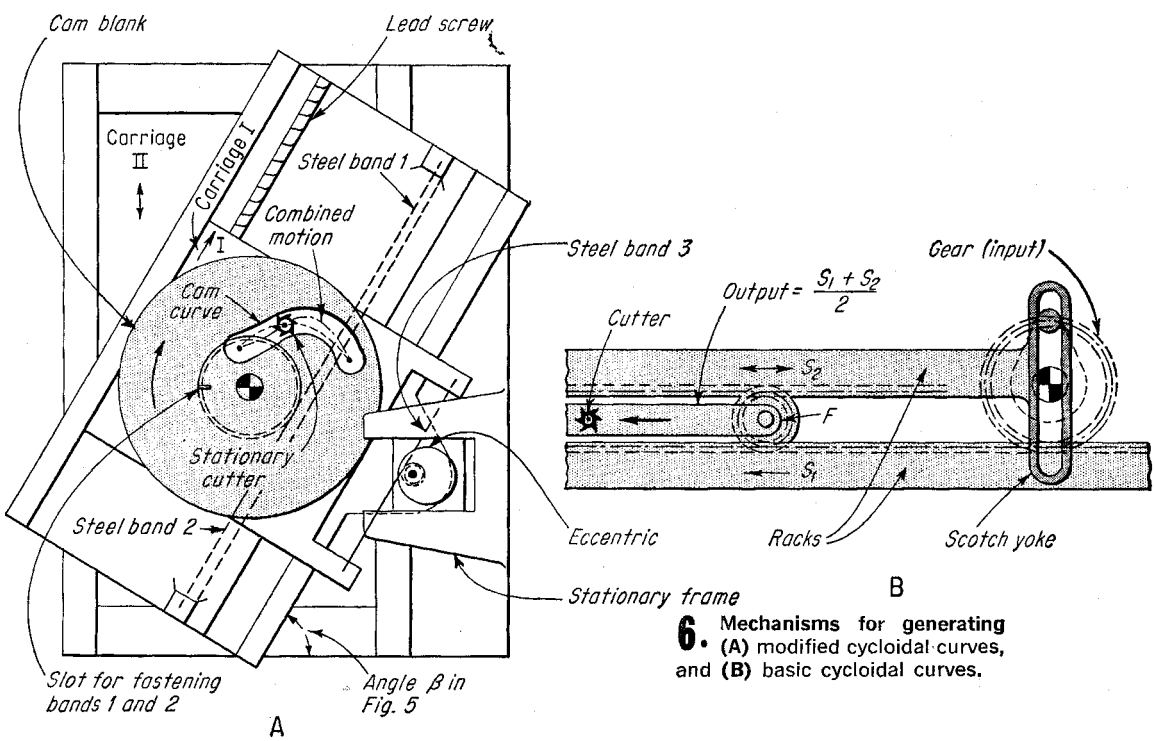
This method is frequently better than copying from a drawing or filing the scallops away from a cam where a great number of points have been calculated to determine the cam profile.



7. Technique for machining circular-arc cams. Radaii r_2 and r_5 are turned on lathe; hardened templates added to r_1, r_3, and r_4 for facilitating hand filing.

Compensating for dwells

One disadvantage with the previous generating devices is that, with the exception of the circular cam, they cannot include a dwell period within the rise-and-fall cam



6. Mechanisms for generating (A) modified cycloidal curves, and **(B)** basic cycloidal curves.

cycle. The mechanisms must be disengaged at the end of rise and the cam rotated in the exact number of degrees to where the fall cycle begins. This increases the inaccuracies and slows down production.

There are two devices, however, that permit automatic machining through a specific dwell period: the double-geneva drive and the double eccentric mechanism.

Double-genevas with differential

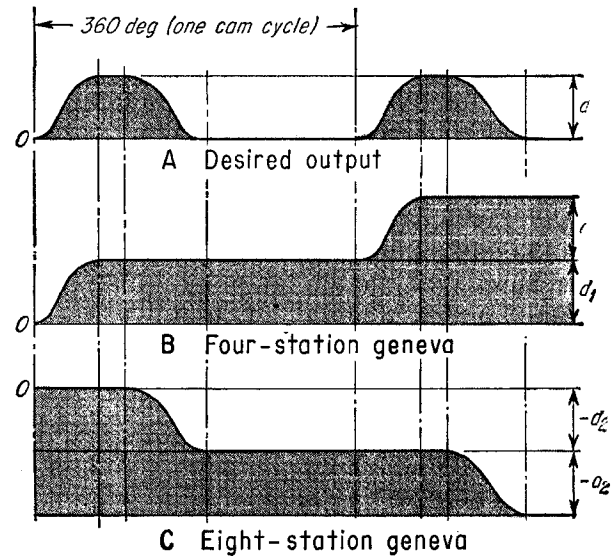
Assume that the desired output contains dwells (of specific duration) at both the rise and fall portions, as shown in Fig 8A. The output of a geneva that is being rotated clockwise will produce an intermittent motion similar to the one shown in Fig 8B—a rise-dwell-rise-dwell . . . etc, motion. These rise portions are distorted simple-harmonic curves, but are sufficiently close to the pure harmonic to warrant use in many applications.

If the motion of another geneva, rotating counterclockwise as shown in (C), is added to that of the clockwise geneva by means of a differential (D), then the sum will be the desired output shown in (A).

The dwell period of this mechanism is varied by shifting the relative position between the two input cranks of the genevas.

The mechanical arrangement of the mechanism is shown in Fig 8D. The two driving shafts are driven by gearing (not shown). Input from the four-star geneva to the differential is through shaft *3;* input from the eight-station geneva is through the spider. The output from the differential, which adds the two inputs, is through shaft *4.*

The actual device is shown in Fig 8E. The cutter is fixed in space. Output is from the gear segment which rides on a fixed rack. The cam is driven by the motor which also drives the enclosed genevas. Thus, the entire device reciprocates back and forth on the slide to feed the cam properly into the cutter.



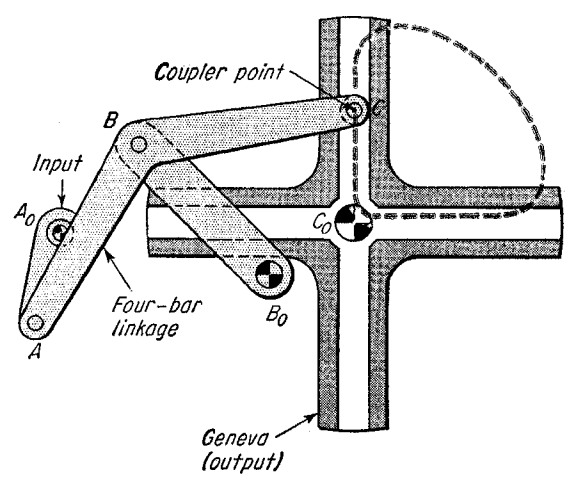
8. Double genevas with differential for obtaining long dwells. Desired output characteristic **(A)** of cam is obtained by adding the motion **(B)** of a four-station geneva to that of **(C)** eight-station geneva. The mechanical arrangement of genevas with a differential is shown in **(D);** actual device is shown in **(E).** A wide variety of output dwells **(F)** are obtained by varying the angle between the driving cranks of the genevas.

Genevas driven by couplers

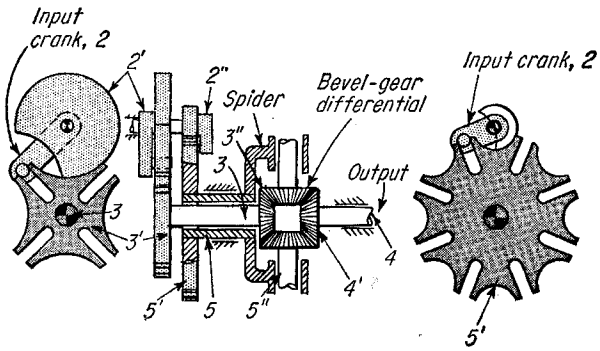
When a geneva is driven by a constant-speed crank, as shown in Fig 8D, it has a sudden change in acceleration at the beginning and end of the indexing cycle (as the crank enters or leaves a slot). These abrupt changes can be avoided by employing a four-bar linkage with coupler in place of the crank. The motion of the coupler point *C* (Fig 9) permits smooth entry into the geneva slot.

Double eccentric drive

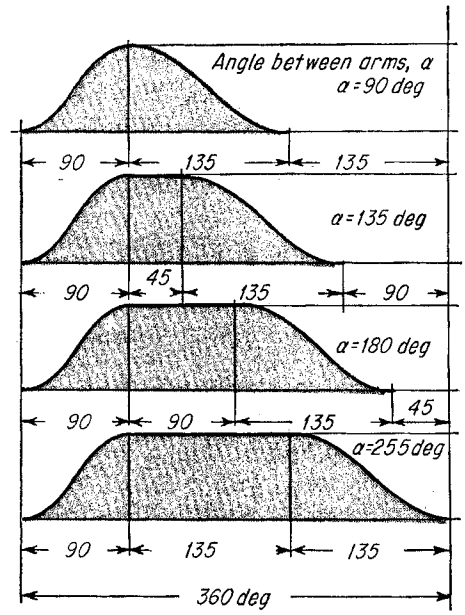
This is another device for automatically cutting cams with dwells. Rotation of crank *A* (Fig 10) imparts an oscillating motion to the rocker *C* with a prolonged dwell at both extreme positions. The cam, mounted on the rocker, is rotated by means of the chain drive and thus is fed into the cutter with the proper motion. During the dwells of the rocker, for example, a dwell is cut into the cam.



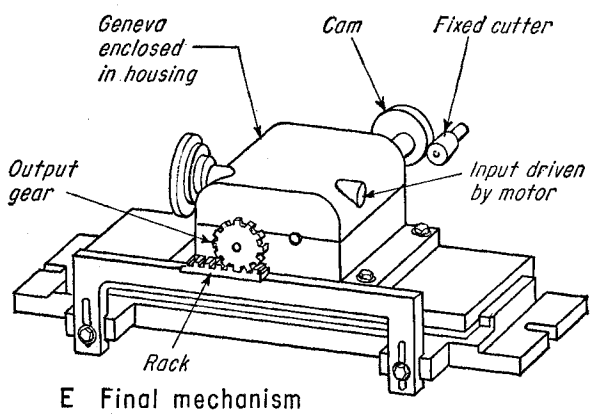
9. Four-bar coupler mechanism for replacing the cranks in genevas to obtain smoother acceleration characteristics.



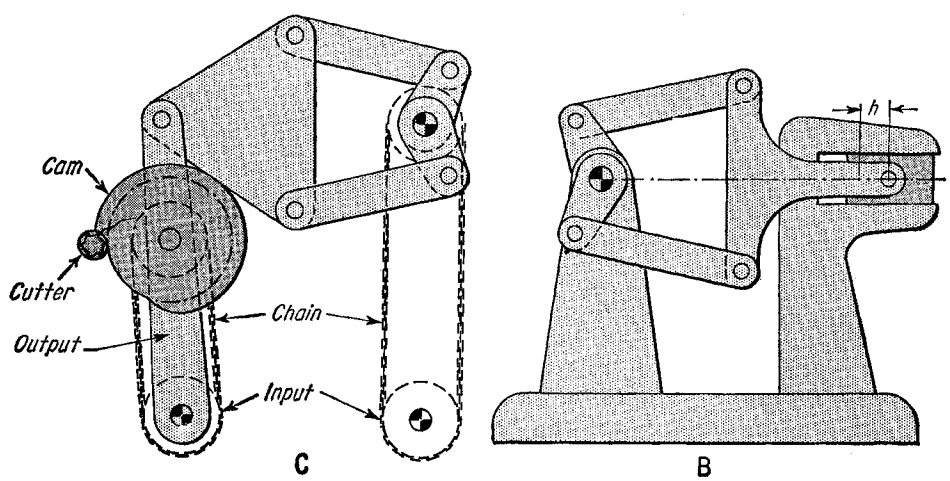
D Double geneva with differential



F Various dwell resultants



E Final mechanism



C B

10. **Double eccentric drive** for automatically cutting cams with dwells. Cam is rotated and oscillated, with dwell periods at extreme ends of oscillation corresponding to desired dwell periods in cam.

Balance Grooved Cams

A quick analytical method for computing the rim cut needed to balance a cam, and a layout method for refining the results.

R. F. Koen

SYMBOLS

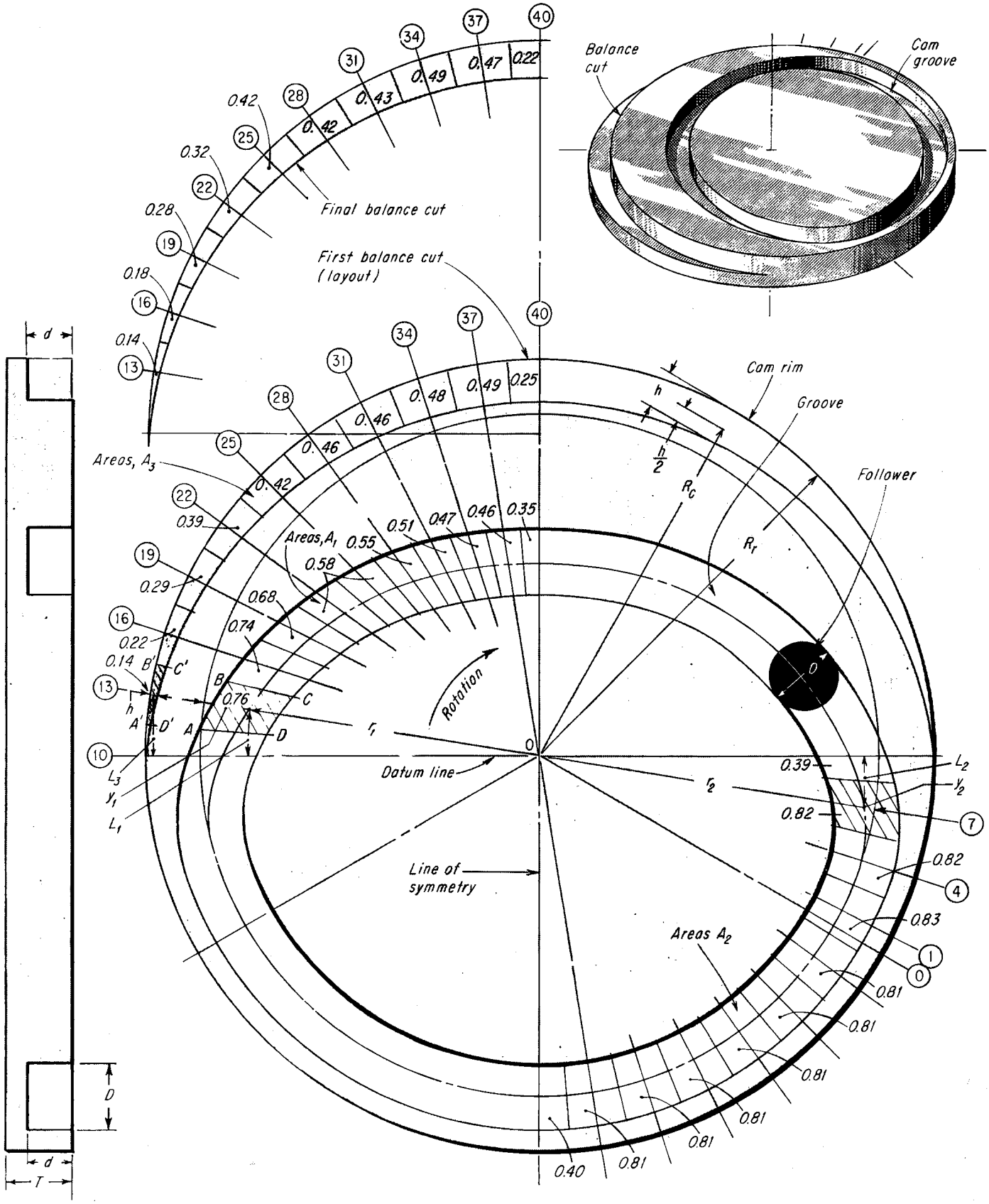
A = area of segment, in.^{2}
C = constant $= 1.5D/R_r$
d = depth of cam groove, in.
D = cam follower diameter, in.
E = unbalance moment, in.^{3}
F = centrifugal force, lb
g = gravitational constant, 32 ft/sec^2
h = radial width of periphery balance cut at specific cam angle, in.
L = perpendicular distance from vertical axis of cam to center of area segment
M = mass, lb-sec^2/in.
n = speed of rotating body, rpm
r_{max} = maximum radius of cam groove, in.
r_1 = radius of groove centerline on lighter side, in.
r_2 = radius of groove centerline on heavier side (diametrically opposite to r_1) in.
R_r = radius to rim, in.
R_c = radius to center of balance cut ($R_c = R_r - h/2$)
T = thickness of cam, in.
V = velocity, in./sec
W = weight of body, lb
y_1 = feed-in dimension (radial displacement of cam follower) on lighter side of cam (from cam displacement tables), in.
y_2 = feed-in dimension on heavier side of cam, in.
ρ = density of cam material, lb/in.^{3}

WHEN a machine is redesigned to increase its productivity, the first approach is to boost its operating speed. What you invariably run up against, however, is the possiblity of destructive vibrations induced by unbalance or by velocity changes in the rotating parts. Frequently the culprits are cams—common machine elements —which create direct or indirect forces.

Direct forces are those induced by the cam profile in accelerating the cam follower. We are not concerned here with profile design (see Editor's Note for articles on this subject) but with indirect forces developed by the inherent unbalance of the cam, in particular, a grooved cam. When the groove is machined, Fig 1, the plate becomes unbalanced with respect to rotation about the center, point O. The solution recommended here is to nullify the unbalance by removing a portion of the rim. This "balance cut" (shown in color) is machined to the same depth as the depth of the groove—or the plate can be cast without the undesirable rim sector.

Equations are presented here which determine, approximately, the varying radial width of the cut, h. A layout procedure is included for checking the amount of balance error that will remain if the original layout for the balance cut is followed through. A tabular method then pinpoints the additional modifications the original cut requires. The result is an accurately balanced cam, as attested by the performance of dozens of cams we have machined and then checked with balancing instruments.

There are no restrictions as to the



1 . . GROOVE CAM WITH FIRST AND FINAL BALANCE-CUT LAYOUTS.

type of cam curve that can be used with this method. However, as stated above, the method is directly applicable to groove cams only. Plate cams —those with the cam profile on the rim—cannot be balanced in the same manner (by simply removing material from the rim) because this will obviously obliterate part of the working profile. Lead inserts and drilled balance holes are employed for this purpose and it is likely that they can be accurately located by suitably modifying the method given here. Such a modification is presently under study.

In applying this method it is assumed that:

1) The balance cut on the rim of the cam is relatively small and therefore the difference between the rim radius, R_r, and the radius to the center of the balance cut will be small.

2) The section to be cut can be considered a series of straight lines.

3) The difference between the diameter of the cam follower and the width of the groove at the section cut is insignificant.

Empirical equations

For static equilibrium, the summation of moments around O must be equal to zero; hence

$$r_1D + R_rh - r_2D = 0 \qquad (1)$$

See Box on page 64 for definition of symbols. From Eq 1, the radial depth of the balance cut, h, is

$$h = \frac{D(r_2 - r_1)}{R_r} \qquad (2)$$

If y is the cam-follower radial displacement given in cam-displacement tables—or the feed-in dimension for machining the groove—then on the lighter side of the cam (the side where the groove is closest to the cam center):

$$r_1 = r_{max} - y_1$$

and on the heavier side (diametrically opposite the lighter side):

$$r_2 = r_{max} - y_2$$

Substituting these values into Eq 2 results in

$$h = \frac{D}{R_r}(y_1 - y_2) \qquad (3)$$

However, Eq 3 has been based on a two-dimensional analysis. It does not take into consideration moments either above or below the plane where the bottom of the cut is made (which is at the same level as the bottom of the groove). By comparing the h values of correctly balanced cams with the h values obtained from Eq 3 it was found that the measured h values were approximately 50% greater than the calculated values. Hence Eq 3 is modified to read:

$$h = \frac{1.5D}{R_r}(y_1 - y_2) \qquad (4)$$

The ratio $1.5D/R_r$ is constant for any given cam; hence letting

$$C = 1.5D/R_r \qquad (5)$$

then

$$h = C(y_1 - y_2) \qquad (6)$$

Layout method

The advantage of employing Eq 6 is that you can directly and quickly determine the h values from the cam displacement tables. These h values usually provide suitably balanced cams. For more critical applications, however, the layout procedure below will produce more accurately balanced cams. We have checked the results of the method with our balancing machines. This method is applied after the y values from the displacement table and the h values from Eq 6 are determined. Referring to Fig 1:

1) Locate the datum line. This line (line 10-0) is perpendicular to the line of maximum unbalance (in the case of Fig 1, the line of symmetry).

2) Lay out equal angular segments on the cam groove and on the rim balance cut.

3) Measure the area of each segment by a planimeter (areas A_1, A_2, and A_3).

4) Locate the center of each area by construction.

5) Measure the perpendicular distances to the centers from the datum line (distances L_1, L_2, and L_3).

6) Sum up the area moments on the balanced and unbalanced sides of the cam (ΣA_1L_1, ΣA_2L_2 and ΣA_3L_3).

7) Check for balance. If rim cut is of proper dimensions then the following equation holds true:

$$\Sigma A_2L_2 - \Sigma A_1L_1 = \Sigma A_3L_3$$

8) If this equation is not satisfied then constant C must be modified as follows:

$$C_1 = C\left(\frac{A_2L_2 - A_1L_1}{A_3L_3}\right) \qquad (7)$$

9) Compute new h values by employing the equation

$$h_1 = C_1(y_1 - y_2) \qquad (8)$$

A more accurate balance cut can now be made by using these new values for h. Before actually machining the cut, the new balance curve can be plotted on the cam and then checked by repeating the above procedure. This can lead to a still more accurate balance; however, cams cut to first-generation h_1 values have proved highly satisfactory.

Design example

A shop layout drawing of an actual cam with $D = 1.002$ in., $R_r = 5.875$ in., and $d = 0.687$ in. is shown in Fig 2. It is to be made of meehanite ($\rho = 0.260$ lb/in.^{3}). Cam speed n is equal to 400 rpm. The cam displacement diagram is shown above the layout drawing. Cam displacement values are given in Table I.

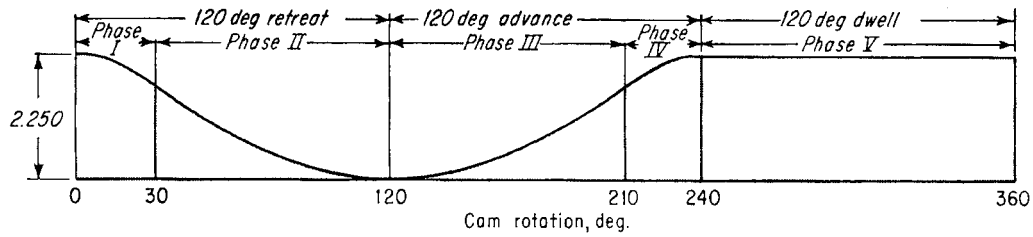
From Eq 5

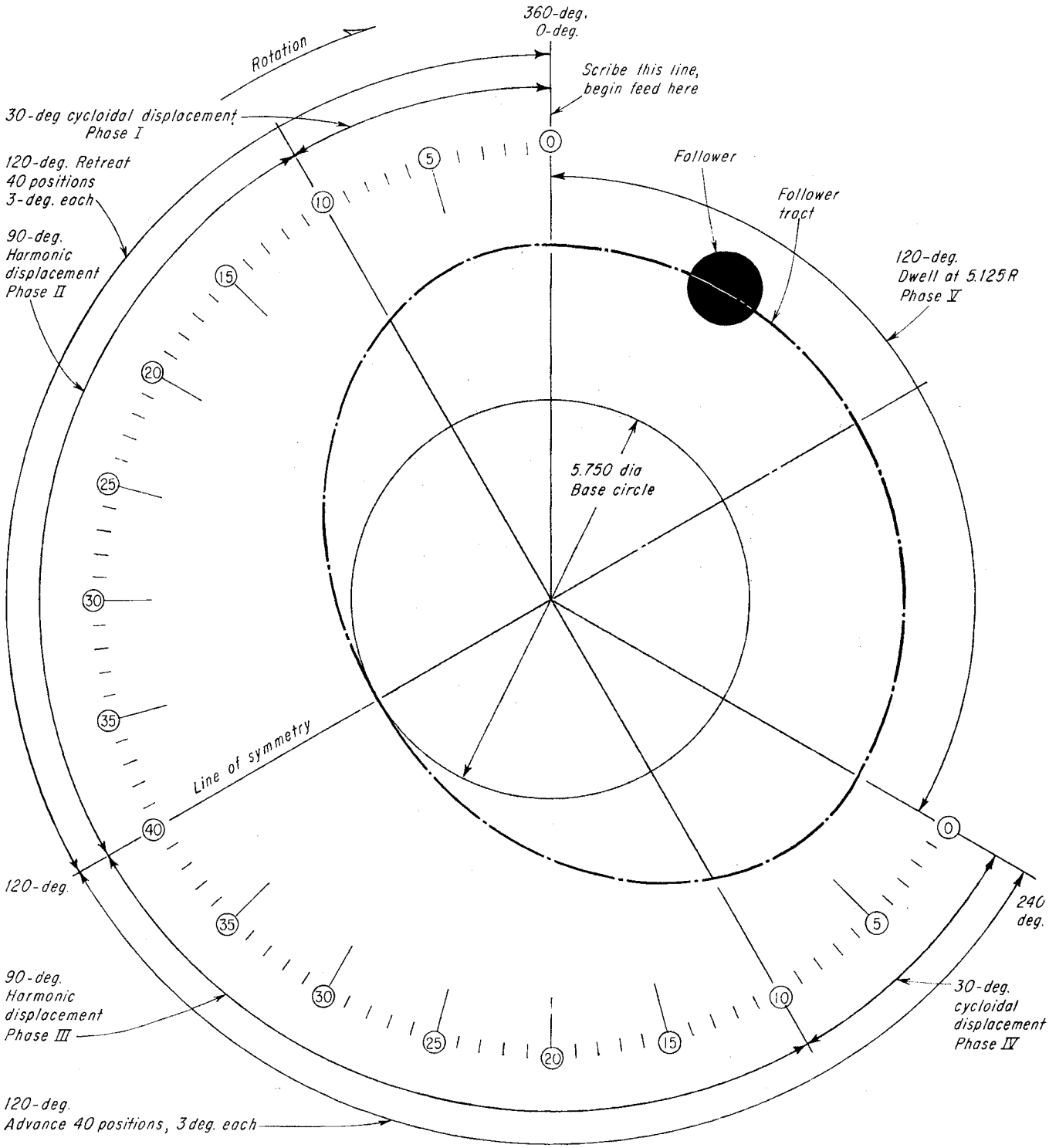
$$C = 1.5(1.002)/5.875 = 0.255$$

1) Determine the y-values per Table I. Values for only half the cam in this example need to be determined

TABLE I. . CAM DISPLACEMENT DATA

POSITION	ANGLE		DISPLACEMENT, IN.
0	0	240	0.000
1	3	237	0.001
2	6	234	0.006
3	9	231	0.020
4	12	228	0.045
5	15	225	0.085
6	18	222	0.139
7	21	219	0.207
8	24	216	0.286
9	27	213	0.374
10	30	210	0.467
11	33	207	0.560
12	36	204	0.653
13	39	201	0.746
14	42	198	0.838
15	45	195	0.928
16	48	192	1.018
17	51	189	1.106
18	54	186	1.192
19	57	183	1.276
20	60	180	1.358
21	63	177	1.438
22	66	174	1.515
23	69	171	1.589
24	72	168	1.660
25	75	165	1.728
26	78	162	1.792
27	81	159	1.853
28	84	156	1.909
29	87	153	1.962
30	90	150	2.010
31	93	147	2.056
32	96	144	2.096
33	99	141	2.132
34	102	138	2.163
35	105	135	2.189
36	108	132	2.211
37	111	129	2.228
38	114	126	2.241
39	117	123	2.248
40	120	120	2.250





2 . . CAM DISPLACEMENT DIAGRAM AND SHOP LAYOUT DRAWING.

TABLE II. . BALANCING-ERROR ANALYSIS

1	2	3	4	5	6	7	8	9	10
	Computing the balancing cut					For displacement groove			
Cam position	Displacement, y_1 (from Table I)	Opposite position number	Displacement, y_2 (from Table I)	Unbalance y_1-y_2	Balance cut, h	Area A_1	Distance L_1	Product A_1L_1	Area A_2
40	2.250	—	0	2.250	0.574	0.35	2.90	1.015	0.40
37	2.228	—	0	2.227	0.568	0.46	2.89	1.330	0.81
34	2.163	—	0	2.163	0.552	0.47	2.84	1.335	0.81
31	2.056	—	0	2.056	0.524	0.51	2.75	1.403	0.81
28	1.909	—	0	1.909	0.487	0.55	2.62	1.441	0.81
25	1.728	—	0	1.728	0.441	0.58	2.41	1.399	0.81
22	1.515	—	0	1.515	0.386	0.58	2.13	1.235	0.81
19	1.276	1	0.001	1.275	0.325	0.68	1.75	1.190	0.83
16	1.018	4	0.045	0.973	0.248	0.74	1.28	0.948	0.82
13	0.746	7	0.207	0.539	0.137	0.76	0.68	0.516	0.83
									$\Sigma A_2L_2=11.812$

because the cam is symmetrical about the centerline. Diametrically opposite positions, however, do not have identical displacement values. Diametric displacement values can be determined from Table I as follows:

Position 13 (39 deg of cam):

$$y_1 = 0.746 \text{ in.}$$

Position 7(39 + 180 = 219 deg of cam):

$$y_2 = 0.207 \text{ in.}$$

Record the difference $(y_1 - y_2)$ in column 5 of Table II.

2) Calculate the h values per Eq 6. For example, the depth of the balance cut at position 13 is

$$h = 0.255(0.746 - 0.207) = 0.137 \text{ in.}$$

Hence multiply the values in column 5 by 0.255 to obtain column 6. If a diametrically opposite side falls between two cam displacement positions then the exact displacement value is determined by interpolation.

3) Locate the line of maximum unbalance. This will be where the h value is maximum—in this problem at position 40. The datum line, or *vertical axis,* as it is called, will be perpendicular to the line of maximum unbalance.

4) Lay out equal angular segments —both on the cam groove and on the rim balance cut.

5) Measure each area segment with a planimeter on the cam groove and on the balance cut. Record these values in Table II, columns 7, 10 and 13. Thus, for position 13: $A_1 = 0.76$, $A_2 = 0.83$, $A_3 = 0.14$ in.^{2}

6) Locate the center of each measured area by construction. Measure normal distances from area center to vertical axis and record in Table II. For position 13: $L_1 = 0.68$, $L_2 = 0.77$, $L_3 = 0.90$ in.

7) Multiply each area segment by its distance value. For example, position 13: $A_1L_1 = (0.76)(0.68) = 0.516$ in.^{3}

8) Sum up each column of AL values. Thus

$$\Sigma A_1L_1 = 11.812$$
$$\Sigma A_2L_2 = 26.399$$
$$\Sigma A_3L_3 = 15.367$$

9) Check for balance. Determine the magnitude of unbalance force at the operating speed of the cam. If the cam speed is relatively slow, and so develops small unbalance forces in operation, it may not be advisable to go to the expense of making a balance cut. Without a balance cut, the error, E, is equal to

$$E_{\text{unbalanced}} = \Sigma A_2L_2 - \Sigma A_1L_1 = 26.399 - 11.812 = 14.587 \text{ in.}^3$$

With a balance cut the error is

$$E_{\text{balanced}} = \Sigma A_2L_2 - \Sigma A_1L_1 - \Sigma A_3L_3 = 26.399 - 11.812 - 15.367 = 0.780$$

The basic equation for centrifugal force is

$$F = \frac{MV^2}{R}, \text{ lb}$$

Converting to in.-lb-rpm units

$$F = \frac{W(2\pi Rn)^2}{32.2(12)(3600)R} = 28.416 \times 10^{-6}\,(WRn^2)$$

Weight W of a sector is equal to $Ad\rho$. Also R can be replaced by L. Since error $E = AL$, the equation for the centrifugal force due to the summation of the unbalanced masses becomes

$$F = 28.416 \times 10^{-6}(d\rho n^2)E = (28.416)(10^{-6})(0.687)(0.260)(400)^2E$$

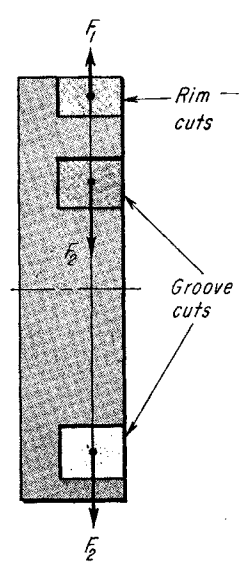
$$F = 0.81E$$

Without a balance cut the centrifugal force will be

$$F_{\text{unbalanced}} = (0.81)(14.587) = 11.8 \text{ lb}$$

With a balance cut

$$F_{\text{balanced}} = (0.81)(0.780) = 0.63 \text{ lb}$$



3. . FOR PERFECT BALANCE, rim cut must be same depth

11	12	13	14	15	16	17	18	19	20
For opposite groove		For balancing cut			Final corrected balancing cut				
Distance L_2	Product A_2L_2	Area A_3	Distance L_3	Product A_3L_3	Unbalanced y_1-y_2	Balanced cut, h_1	Area A_3	Distance L_3	Product A_3L_3
5.12	2.045	0.25	5.58	1.395	2.250	0.544	0.22	5.59	1.230
5.06	4.100	0.49	5.53	2.710	2.227	0.539	0.47	5.53	2.600
4.87	3.943	0.48	5.33	2.560	2.163	0.523	0.49	5.33	2.610
4.56	3.700	0.46	5.08	2.340	2.056	0.498	0.43	5.02	2.108
4.15	3.360	0.46	4.55	2.095	1.909	0.462	0.42	4.55	1.980
3.63	2.940	0.42	4.00	1.680	1.728	0.418	0.42	4.00	1.680
3.01	2.439	0.39	3.38	1.320	1.515	0.367	0.32	3.38	1.082
2.33	1.935	0.29	2.59	0.751	1.275	0.309	0.28	2.59	0.725
1.58	1.298	0.22	1.77	0.390	0.973	0.235	0.18	1.77	0.318
0.77	0.639	0.14	0.90	0.126	0.539	0.130	0.14	0.90	0.126
$\Sigma A_1L_1=26.399$			$\Sigma A_3L_3=15.367$						$\Sigma A_3L_3=14.459$

10) Correct the initial balance-cut layout. With a balance cut on the rim, the unbalance force is only 0.63 lb. Because this force is small it is probably not necessary to make a correction in the C value and recalculate the h value. The correction is computed, however, for illustration purposes. From Eq 7:

$$C_1 = 0.255\left(\frac{26.399 - 11.812}{15.267}\right)$$

$$C_1 = 0.242$$

11) Recalculate the h values per corrected constant C. Example, for position 13, the previous value of $h = 0.137$ in column 5 will be replaced by the value

$$h_1 = 0.539(0.242) = 0.130$$

This is entered into column 17.

12) Lay out the new rim balance cut. Use corrected h values.

13) Determine new values for A_3 and L_3. These are entered into columns 18, 19 and 20. Thus

$$\Sigma A_3L_3 = 14.459$$

The final balance cut is shown at top of Fig 1.

14) Recheck for balance

$$\Sigma A_2L_2 - \Sigma A_1L_1 - \Sigma A_3L_3 = 0$$

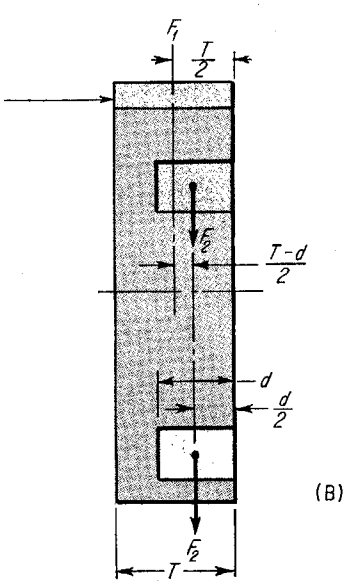
$$26.399 - 11.812 - 14.459 = 0.128 \text{ (unbalanced)}$$

The recheck for balance still does not yield perfect results; however, inaccuracies in methods of measurement to determine balance values easily account for this small unbalance. The centrifugal force will be

$$F = 0.81(0.128) = 0.01 \text{ lb.}$$

Because the balance cut is made on the rim to the exact depth of the cam groove, the cam is dynamically as well as statically balanced. For example, in Fig 3A, a balance cut made to the depth of the cam groove places the centrifugal force, F_1, due to the balance cut in the same normal plane as the unbalance force, F_2, due to the groove. There is no dynamic couple acting on the shaft.

If, however, the balance cut is made the full width of the cam blank, as in (B), the centrifugal force F_1, due to the balance cut, and the centrifugal force F_2 acting at the midpoint of the groove will not line up. This produces a dynamic couple $F_2\ (T - d)2$.



as groove. Full-width rim-cut causes unbalance moments.

The cam in (B) is statically but not dynamically balanced.

The machining of a balance cut is simplified by the fact that a radius usually can be found which will pass through most of the h points. If this is possible, the cam can be chucked off-center on a lathe and machined quite simply. If one radius does not suffice the balance cut can be made on a cam miller in a manner similar to the machining of the cam groove.

A full-width balance cut, Fig 3B, is sometimes made to facilitate machining; for example, a cut can be made with a band saw on a scribed line. In such cases, Eq 5 should be modified to read

$$C = \frac{1.5\ Dd}{R_rT}$$

The cam, of course, will not be dynamically balanced.

Computer and N/C Simplifies Cam Design

Complex, "exotic" cam curves can now be confidently specified without tedious calculations. And numerical control assures accurate machining of the chosen contours.

Nicholas P. Chironis

New technique produces wide variety of shapes, including barrel and cup cams.

A fast computer in tandem with a specially designed numerical-control converter is opening up new versatility in cam design. No longer need the designer fear—and shun—the complex, "exotic" cam curves in favor of the older and more familiar ones.

The more complex curves entail long, tedious calculations to plot a series of coordinates for the contours. Moreover, machining such contours is slow and expensive unless numerical control can be used.

Making it easy. With the equipment and design procedure developed by Cam Technology, Inc., Elmsford, N.Y., a Cam Tech engineer can take a cam drawing (photos left) and read off the desired displacement requirements of the follower. He punches the dimensional data into the computer, plus the type of cam curve he judges best for the application, and out comes a long punched tape completely designing the cam.

This paper tape is fed into the special N/C converter, which can be wheeled up to operate any standard high-precision jig borer and jig grinder. With this technique, mathematically precise contours and unusual cam configurations have been made to tolerances in the ten-thousandths of an inch.

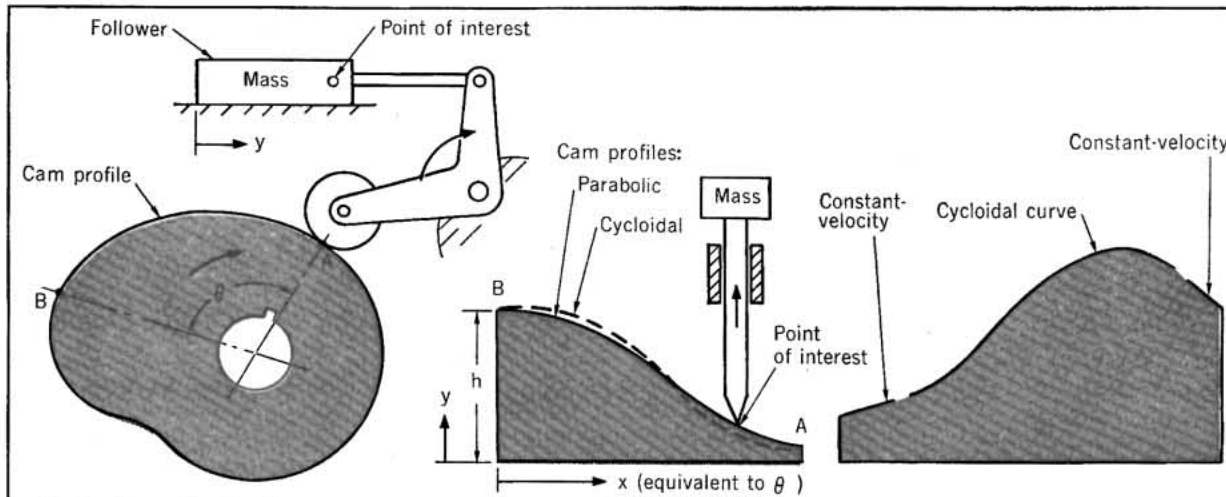
The function of a cam, of course, is to displace a follower by a specified linear or angular distance while the cam itself rotates a specific amount. Simple as this may seem, the variety of possible cam types and profiles is almost endless. Each design engineer, though, has his own preferences.

To simplify the drawing specifications, Theodore Weber, Cam Tech's president, set up the computer-converter system to solve a cam problem in three phases: (1) operational requirements, (2) cam profile selection, and (3) linkage conversion.

Operational specifications. The first item is simply the designer's basic requirements, relating the displacement of a point in the follower linkage to camshaft rotation.

In the drawing (p 52), a machine part must move a distance y while the cam rotates an angle θ. This motion is pictured in linearized form as moving a plunger vertically the distance y.

For most cams, only the coordinates of the beginnings and ends of



1. Designer's displacement requirements **2. Linearization of the cam profile** **3. Matching cam curve to end requirements**

segments need be given, such as the location of the points *A* and *B*. For "function" type cams, in which a certain mathematical relationship between input and output must be maintained, the equation or equations governing this relationship must also be specified.

Curve selection. Many curves can be employed to move the plunger for the portion of the cam profile, *A*—*B*. Some are common, some are less known. To the eye they look the same—but they produce very different dynamic effects on the follower. One curve may cause high acceleration forces, another may induce an abrupt change in acceleration, and still another may cause dangerous vibrations.

Cam Tech programmed into the computer dozens of cam curves, so the designer need only specify the type of curve he prefers. Moreover, all the curves have been converted into more involved generalized forms to fit any type of slope termination. A cycloid curve, for example, can be asked to match, with ease, constant-velocity portions (drawing, above).

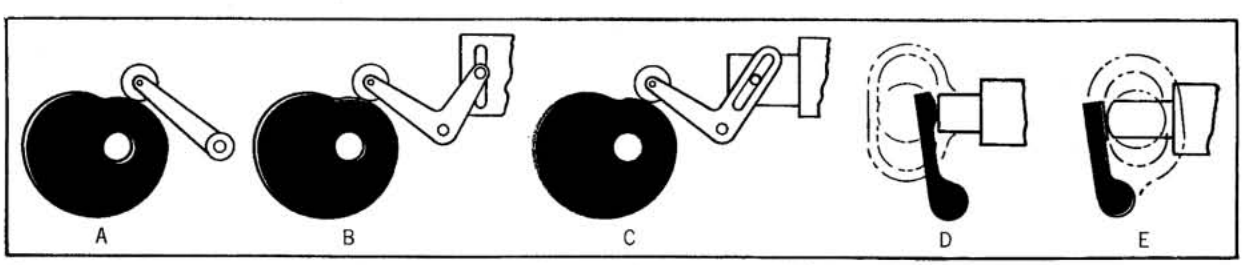
Linkage conversion. The conversion of linearized data to the specialized linkage arrangement that a designer may require involves even more complex calculations than those for the cam profile itself. The endless variety of linkage arrangements between cam and point of application would seem to make the computerization of such a conversion a hopeless task.

However, experience has shown that the vast majority of linkages fall into one of four classifications: the three shown in the drawing below as Types A, B, and C and the connecting-rod type in the drawing above.

Many seemingly unrelated types of follower linkage will fall into one of the above categories. For instance, if a flat face on the follower-arm branch pushes the curved surface on a slider (Type D), it is only necessary to complete the circle of that curved surface to note that this shape comes under Type B. Similarly, a curved face that pushes a straight face on the slider (Type E) falls under Type C.

Further transformations are necessary when the follower diameter is not also a feasible cutter diameter. The follower may, for instance, be flat, or the contour may be specified at the cam surface. Parallel-curve transformation is then used to define a feasible cutter path.

Pick a curve. Cams are often the limiting factor in determining how fast a machine can be operated continuously. A poorly selected profile can produce high dynamic loads and vibration at high speeds that



Computer must transform basic curve into desired follower-linkage system. Majority of linkages fall into these types.

can literally rip any machine apart.

Parabolic curve. This will produce the lowest maximum acceleration. Its equation, for the first half of the follower motion, y, is:

$$y = 2h\left(\frac{\theta}{\beta}\right)^2$$

where h = maximum rise of follower, in.
θ = cam-angle rotation for follower displacement, y
β = total cam-angle rotation

The shape of its acceleration is rectangular (drawing, below) with an abrupt change from positive acceleration to negative acceleration (deceleration). This change, sometimes called "infinite jerk," can induce transient shock waves that are especially destructive if there is any looseness in the system.

The parabolic curve, therefore, should be limited to low-speed application. It is still specified by some machine tool builders, but it is Weber's recommendation that this curve be avoided whenever possible.

Simple harmonic curve. This is both easy to calculate and to construct graphically. It eliminates the midpoint transient of the parabolic (at a cost of a 23% higher acceleration), but it unfortunately also produces an abrupt change in acceleration in the beginning and end of its cycle. With the new computer techniques, fewer designers should be calling for this type of curve. Its equation is:

$$y = \frac{h}{2}\left[1 - \cos\left(\frac{\pi\theta}{\beta}\right)\right]$$

The cubic curve shown in the diagram also suffers from an abrupt change in the ends of the cycle.

Cycloidal curve. This is a favorite among many designers because it has no abrupt changes in acceleration and gives low vibration, noise, and shock. Many designers formerly shunned this curve because it called for a higher degree of machining accuracy than did those previously described. To a mathematician, its equation reveals its inherent complexity:

$$y = h\left[\frac{\theta}{\beta} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\beta}\right)\right]$$

Third harmonic curve. This is a curve derived by Weber that can be tailored to operate below the natural frequency of the follower system and thus can avoid resonance. It is actually a modified cycloidal curve, obtained by introducing a small third-harmonic component in the cycloidal curve to reduce the peak acceleration to 1.28 times the parabolic peak (the cycloidal peak acceleration is about 1.57 times that of the parabolic peak). Its equation is:

$$y = h\left[\frac{\theta}{\beta} - \frac{15}{32\pi}\sin\left(\frac{2\pi\theta}{\beta}\right) - \frac{1}{96\pi}\sin\left(\frac{6\pi\theta}{\beta}\right)\right]$$

This curve is Weber's favorite. It produces smaller dynamic loads than the cycloidal curve (drawing) and smaller vibration amplitudes than all other curves except the cycloidal.

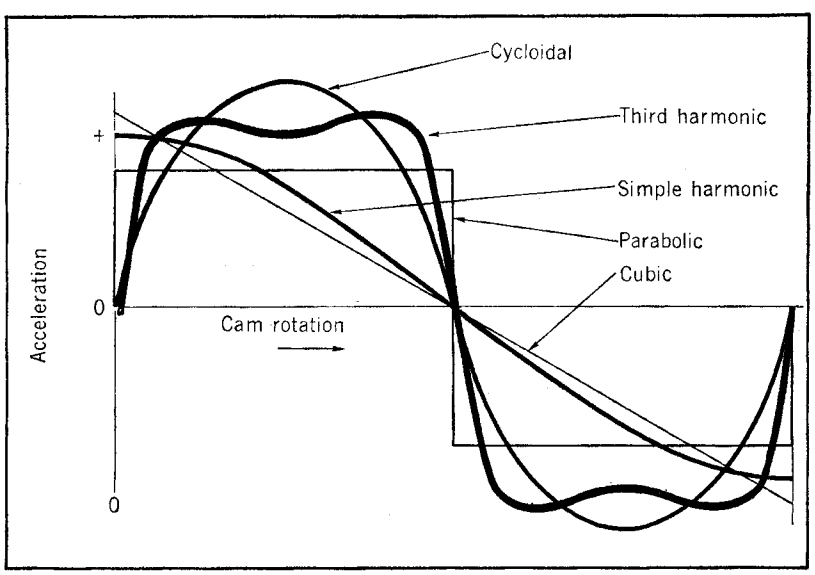
Still other curves are preferred by some engineers, including the so-called "3-4-5" polynomial:

$$y = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

Special N/C controllers. To obtain the needed accuracy for cam fabrication, Weber had to develop his own N/C control center.

Most digital N/C types move from point to point in small steps, with a feedback system that rounds off the error. The analog N/C's produce a smoother curve, but they do not have the accuracy of the digital types.

Weber, therefore, was forced to design a special hybrid system combining the smoothness of the analog system with the accuracy of the digital system. This control system, working in a team with Weber's computer, typically processes one information datum each tenth of a degree, or 3600 data points per complete cam revolution. These data are processed at a rate of five per second, compared with approximately one per second for most N/C systems.



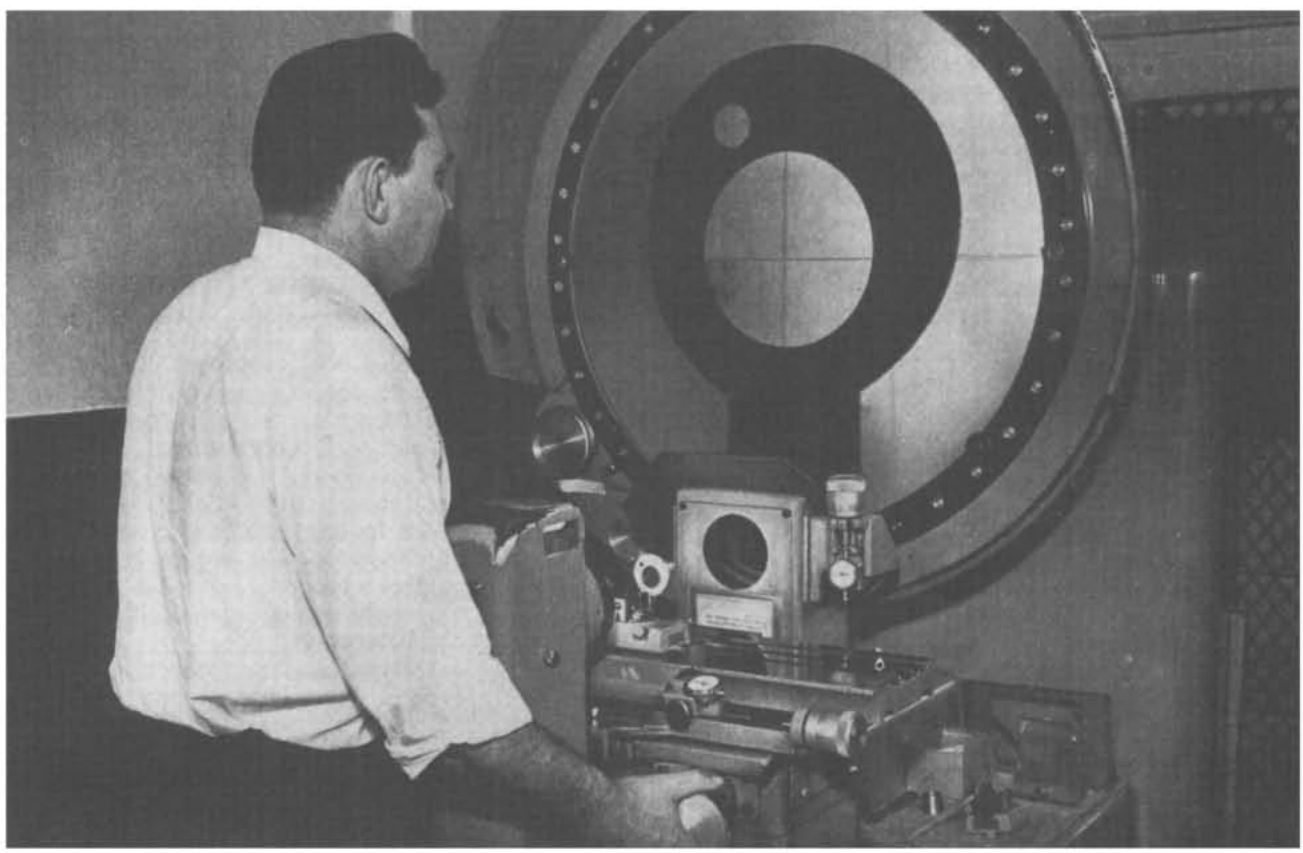
Analysis of five types of cam profile shows that the parabolic, long favored by machine designers for its low maximum-acceleration values, produces an abrupt change in acceleration. So do the cubic and simple harmonic profiles. The cycloidal is smooth, but the little-known third harmonic performs still better.

Theory of Envelopes: Cam Design Equations

These profile and cutter-coordinate equations for six types of cam accept any lift requirements. Included are details on applying the envelope theory to other types.

Dr. Roger S. Hanson & Frederic T. Churchill

ANALYTICAL determinations of cam profiles and cutter coordinates are usually subordinated to graphical techniques because of the voluminous calculations required. In recent years, with the widespread use of high-speed computing equipment, these calculations need no longer be a deterrent. When high-speed, heavy-inertia loads or accurate positioning are design requirements, the designer now has a choice between the analytical approach and the graphical. He is limited only by the ability of present-day machine tools to reproduce the



SHADOWGRAPH CHECKS CAM ACCURACY TO ± 0.0005 IN. MAGNIFICATION 10X.

accurate specifications he has made for the cam profile.

The theory of envelopes has not been employed to any extent in cam design—yet it is a powerful analytical tool. The theory is illustrated here and then applied to the development of profile and cutter-coordinate equations for the six major types of cams:

Flat-face follower cams

- Swinging in-line follower
- Swinging off-set follower
- Translating follower

Roller-follower cams

- Translating follower
- Translating off-set follower
- Swinging follower

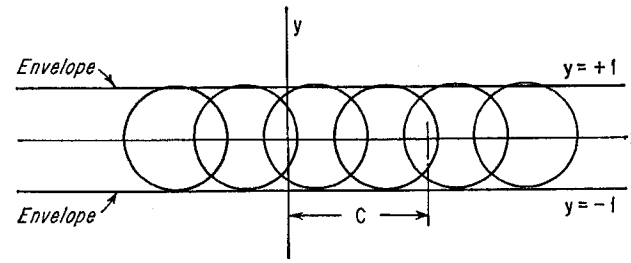
The design equations for these cams (the profile and cutter-coordinate equations) are in a form that accepts any profile curve—such as the cycloidal or harmonic curve—or any other desired input-output relationship. The cutter-coordinate equations are *not* a simple variation of the profile equations, because the normal line at the point of tangency of the cutter and the profile does not continually pass through the cam center. We had need for accurate cutter equations in the case of a swinging flat-face follower cam. The search for the solution led us to employ the theory of envelopes. A detailed problem of this case is included to illustrate the use of the design equations which, in our application, provided coordinates for cutting cams to a production tolerance of ±0.0002 in. from point to point, and 0.002-in. total over-all deviation per cam cycle.

The question will come up whether computers are necessary in solving the design equations. Computers are desirable, and there are many outside services available. Calculations by hand or with a desk calculator will be time consuming. In many applications, however, the manual methods are worth while when judged by the accuracy obtainable. The designer will undoubtedly develop his own short cuts when applying the manual methods.

Application to visual grinding

The design equations offered here can also be put to good advantage in visual grinding. Magnification is limited by the definition of the work blank projected on the glass screen. On a particular visual grinder, the definition is good at a magnification of 30X, although provision is made for 50X. Using Mylar drawing film for the profile, which is to be fixed to the ground-glass screen, a 30X drawing or chart of portions of the cam profile can be made. Best results are obtained by locating the coordinate axis zero near the curve segment being drawn and by increasing the number of calculated points in critical regions to ½ or ¼-deg increments for greater accuracy. (Interpolation between points specified in 2-deg intervals by means of a French curve, for example, suffers in accuracy.) This procedure facilitates checking a cam with a fixture employing a roller, because the position of the roller follower can be specified simultaneously with the profile point coordinates.

The real limitation in visual grinding is the size of ground-glass field and the limited scope of blank profile which can be viewed at one time. If 30X is the magnification for good definition, and the screen is 18 in., the maximum cam profile which can be viewed at one time is 18/30 = 0.60 in. If the layout is drawn 30 times size and a draftsman can measure ±0.010 in., the error in drawing the chart is 0.010/30 = ±0.0003 in. In addition, the coordination of chart with cam blank,



1A . . LINEARLY MOVING CIRCLES

The theory of envelopes is a topic in calculus not always taught in college courses. It is illustrated here by two examples, before we proceed to apply to it cam design.

The envelope can be defined this way: If each member of an infinite family of curves is tangent to a certain curve, and if at each point of this curve at least one member of the family is tangent, the curve is either a part or the whole of the envelope of the family.

Linearly moving circle

As the first example of envelope theory, consider the equation

$$(x - c)^2 + (y)^2 - 1 = 0 \qquad (1)$$

This represents a circle of radius 1 located with its center at $x = c$, $y = 0$. As c is varied, a series of circles are determined—the family of circles governed by Eq 1 and illustrated in Fig 1A.

Eq 1 can be rewritten

$$f(x, y, c) = 0 \qquad (2)$$

It is shown in calculus that the slope of any member of the family of Eq 2 is

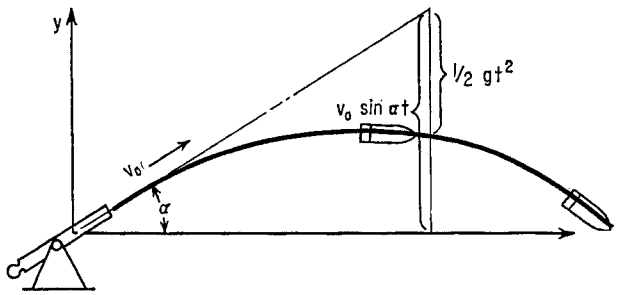
$$\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} \qquad (3)$$

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = 0 \qquad (4)$$

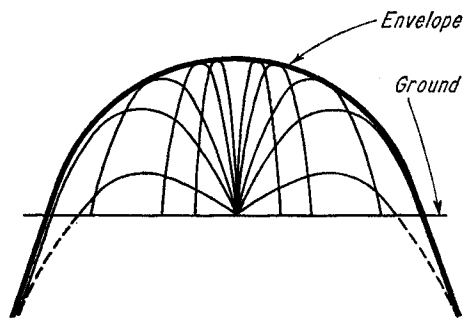
This may be written

$$\frac{\partial f}{\partial x}\,\frac{dx}{dc} + \frac{\partial f}{\partial y}\,\frac{dy}{dc} = 0 \qquad (5)$$

This slope relation holds true for any member of the family. If another curve (the envelope) is tangent to the member of the family at a single point, its slope likewise satisfies Eq 5.



1B . . SHELL TRAJECTORY



1C . . PARABOLIC ENVELOPE OF TRAJECTORIES

It is also shown in calculus that the total differential of Eq 2 is

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial c}\,dc = 0 \tag{6}$$

or

$$\frac{\partial f}{\partial x}\,\frac{dx}{dc} + \frac{\partial f}{\partial y}\,\frac{dy}{dc} + \frac{\partial f}{\partial c} = 0$$

From Eq 5 and 6, the general equation for the envelope is

$$\frac{\partial f(x,y,c)}{\partial c} = 0 \tag{7}$$

The envelope may be determined by eliminating the parameter c in Eq 7 or by obtaining x and y as functions of c. (The point having the coordinates at x and y is a point on the envelope, and the entire envelope can be obtained by varying c.)

Returning to Eq 1 and applying Eq 7 gives

$$\frac{\partial f(x,y,c)}{\partial c} = 2(x - c)\left(\frac{-\partial c}{\partial c}\right) + \frac{\partial y^2}{\partial c} - 0 = 0$$

$$= 2\,(x - c)\,(-1) + 0 - 0 = 0$$

Therefore $x = c$. Substituting this into Eq 1 gives $y = \pm 1$. Thus the lines $y = +1$ and $y = -1$ are the envelopes of the family of Eq 3. This, of course, is evident by inspection of Fig 1.

Shell trajectories

As a second example of envelope theory, consider the envelope of all possible trajectories (the range envelope) of a gun emplacement. If the gun can be fired at any angle α in a vertical plane with a muzzle velocity v_o, Fig 1B, what is the envelope which gives the maximum range in any direction in the given vertical plane? Air resistance is neglected.

The equation of the trajectory is

$$y = x \tan \alpha - \frac{gx^2}{2v_o^2}\,(1 + \tan^2 \alpha) \tag{8}$$

where v_o = muzzle velocity
t = time
g = gravitational constant

Eq 8 is derived as follows:

$$y = v_o \sin \alpha t - \tfrac{1}{2} g t^2 \tag{9}$$

where

$$t = \frac{x}{v_x} = \frac{x}{v_o \cos \alpha} \tag{10}$$

Substituting this value of t into Eq 9 gives

$$y = \frac{v_o \sin \alpha x}{v_o \cos \alpha} - \tfrac{1}{2} g \left(\frac{x}{v_o \cos \alpha}\right)^2 \tag{11}$$

which can be readily put in the form of Eq 8.

Rewriting Eq 8 so that all factors are on one side of the equation:

$$f(x,y,\alpha) = x \tan \alpha - \frac{gx^2}{2v_o^2}\,(1 + \tan^2 \alpha) - y = 0 \tag{12}$$

Thus

$$\frac{\partial f(x,y,\alpha)}{\partial \alpha} = x \sec^2 \alpha \left(1 - \frac{gx \tan \alpha}{v_o^2}\right) = 0 \tag{13}$$

Solving Eq 13 for tan α gives

$$\tan \alpha = \frac{v_o^2}{gx}$$

Eliminating the parameter α by substituting this value of tan α into Eq 8 yields the envelope of the useful range of the gun,

$$y = \frac{v_o^2}{2g} - \frac{gx^2}{2v_o^2} \tag{14}$$

which is a parabola, pictured in Fig 1C.

SYMBOLS

b = y-intercept of straight line
c = linear-distance parameter
e = offset of flat-face or roller follower
f = function notation
g = gravitational constant
$H = r_b + r_f + L$
$J = [(r_b + r_f)^2 - e^2]^{1/2}$
L = lift of follower
$M = \phi - \theta + \Psi$
m = general slope of straight line
$N = \theta - \phi - \Psi$
r_a = distance between pivot point of swinging follower and cam center
r_b = radius of base circle of cam
r_c = radius of cutter
R_c = radius vector from cam center to cutter center. Employed in conjunction with ω
r_f = radius of roller follower
r_r = length of roller-follower arm
t = time
v_o = initial (muzzle) velocity
x, y = rectangular coordinates of cam profile, or of circle or parabola in examples on envelope theory
x_c, y_c = cutter coordinates to produce cam profile
$\frac{d}{dx}$ = total derivative with respect to x
$\frac{\partial}{\partial x}$ = partial derivative with respect to x
α = angle of muzzle inclination in trajectory problem; also angle between x-axis and tangent to cutter contact point
β = maximum lift angle for a particular curve segment = θ_{max}
ω = angular displacement of cutter center, referenced to zero at start of cam profile rise. Employed in conjunction with R_c.
ϵ = angular displacement of cutter, referenced to x-axis, with the cam considered stationary (for specifying polar cutter coordinates); $\epsilon = \tan^{-1}(y_c/x_c)$; also $\epsilon = \omega$ when rise begins at x-axis as in Fig. 7.
θ = cam angle of rotation
ϕ = angular rotation or lift of the follower, usually specified in terms of θ
Ψ = angle between initial position of face of swinging follower, and line joining center of cam and pivot point of follower (a constant)
λ = maximum displacement angle of follower arm

the condition of the machine, and the operator's degree of skill all add some error. In a particular segment, the operator can grind ±0.0003 in., but when the chart and work piece are moved to the next profile segment they must be properly coordinated to take advantage of the grinder's skill and to prevent discontinuities that can affect seriously the dynamic characteristics of the cam.

FLAT-FACE FOLLOWERS

The theory of envelopes is now applied to finding the design equations for cams with flat-face followers. In general:

1) Choose a convenient coordinate system—both rectangular and polar coordinates are given here.

2. Write the general equation of the envelope, involving one variable parameter.

3) Differentiate this equation with respect to the variable parameter and equate it to zero. The total derivative of the variable usually suffices (in place of the partial derivative).

4) Solve simultaneously the equations of steps 2 and 3 either to eliminate the parameter or to obtain the coordinates of the envelope as functions of the parameter.

5) Vary the parameter throughout the range of interest to generate the entire cam profile.

Flat-face in-line swinging follower

Flat-face swinging-follower cams are of the in-line type, Fig 2, if the face, when extended, passes through the pivot point. The initial position of the follower before lift starts is designated by angle ψ. This angle is a constant and can be computed from the equation

$$\Psi = \sin^{-1}\frac{r_b}{r_a}$$

where

r_a = distance between cam center and pivot point measured along x-axis

r_b = radius of base circle of cam

The angular rotation or "lift" of the follower, ϕ, is the output motion. It is usually specified as a function of the cam angle of rotation, θ. Thus θ is the independent variable and ϕ the dependent variable.

A well-known analytical technique is to assume the cam is stationary and the follower moving around it. Varying θ and ϕ and maintaining ψ constant produces a family of straight lines that can be represented as a function of x, y, θ, ϕ. Since ϕ is in turn a function of θ, essentially there is

$$f(x,y,\theta) = 0 \qquad (15)$$

This is the form of Eq 2. Thus to obtain the envelope of this family, which is the required cam profile, one solves simultaneously Eq 15, and

$$\frac{\partial f(x,y,\theta)}{\partial \theta} = \frac{df}{d\theta} = 0 \qquad (16)$$

The first step is to write the general form of the equation of the family. We begin with

$$y = mx + b \qquad (17)$$

Where b is the y-intercept and m the slope. In this case, m is equal to

$$m = -\tan(\phi - \theta + \Psi) \tag{18}$$

Hence

$$y = -\tan(\phi - \theta + \Psi)x + b \tag{19}$$

Also

$$x = r_a \cos\theta$$
$$y = r_a \sin\theta$$

Solving for b results in

$$b = r_a[\sin\theta + \cos\theta \tan(\phi - \theta + \Psi)] \tag{20}$$

Therefore

$$f(x,y,\theta) = y + \tan(\phi - \theta + \Psi)(x - r_a\cos\theta) - r_a\sin\theta = 0 \tag{21}$$

This equation is in the form of Eq 15. It is now differentiated with respect to θ:

$$\frac{df}{d\theta} = \tan(\phi - \theta + \Psi)r_a\sin\theta + (x - r_a\cos\theta)[\sec^2(\phi - \theta + \Psi)]\left(\frac{d\phi}{d\theta} - 1\right) - r_a\cos\theta = 0 \tag{22}$$

For simplification in notation, let

$$M = \phi - \theta + \Psi$$

The rectangular coordinates of a point on the cam profile corresponding to a specific angle of cam rotation, θ, are then obtained by solving Eq 21 and 22 simultaneously. The coordinates are

$$x = r_a\left[\cos\theta + \frac{\cos(\theta + M)\cos M}{\frac{d\phi}{d\theta} - 1}\right] \tag{23}$$

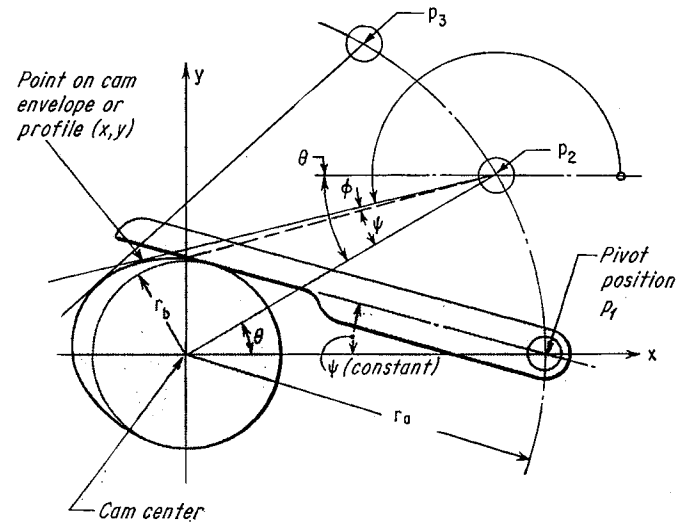
$$y = r_a\left[\sin\theta + \frac{\cos(\theta + M)\cos M}{\frac{d\phi}{d\theta} - 1}\right] \tag{24}$$

As mentioned previously the desired lift equation, ϕ, is usually known in terms of θ. For example, in a computer a cam must produce an input-output relationship of $\phi = 2\theta^2$. In other words, when θ rotates 1 deg, ϕ rotates 2 deg; when θ rotates 2 deg, ϕ rotates 8 deg, etc. Then

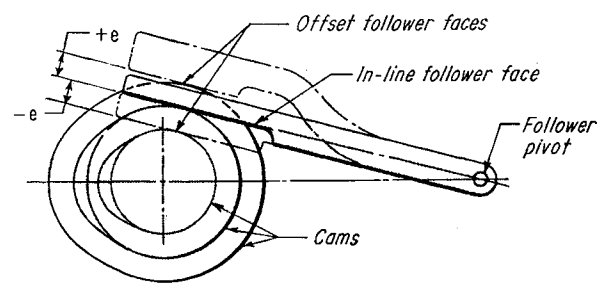
$$\phi = 2\theta^2$$

and

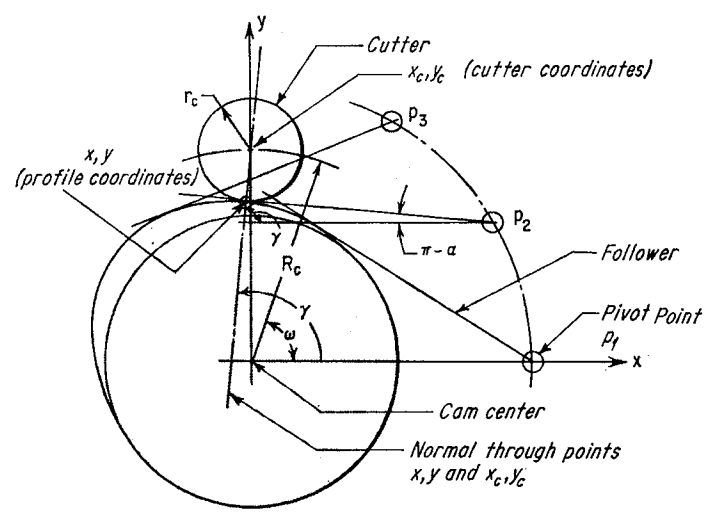
$$\frac{d\phi}{d\theta} = 4\theta$$



2 . . Flat-face swinging-follower cam with line of follower face extending through pivot point.



3 . . Two types of offset flat-face follower.



4 . . Cutter coordinates for flat-face swinging follower.

Substituting the value of ϕ into the equation for m, and the value of $d\phi/d\theta$ into Eq 23 and 24 gives x and y in terms of θ.

Where the lift equation must also meet certain velocity and acceleration requirements (as is the more common case), portions of analytical curves in terms of ϕ, such as the cycloidal or harmonic curves, must be used and matched with each other. A detailed cam design problem of an actual application is given later to illustrate this technique.

Offset swinging follower

The profile coordinates for a swinging flat-faced follower cam in which the follower face is not in line with the follower pivot, Fig 3, are

$$x = r_a\left[\cos\theta + \frac{\cos(\theta + M)}{\frac{d\phi}{d\theta} - 1}\cos M\right] + e\sin M \quad (25)$$

$$y = r_a\left[\sin\theta + \frac{\cos(\theta + M)}{\frac{d\phi}{d\theta} - 1}\sin M\right] + e\cos M \quad (26)$$

where e = the offset distance between a line through the cam pivot and the follower face. Distance e is considered positive or negative, depending on the configuration. In other words, the effect of e in Eq 25 and 26 is to increase or decrease the size of the in-line follower cam. When $e = 0$, Eq 25 and 26 simplify to Eq 23 and 24.

Cutter coordinates

For cam manufacture, the location of the milling cutter or grinding wheel must be specified in rectangular or polar coordinates—usually the latter.

The rectangular cutter coordinates for the in-line swinging follower, Fig 4, are

$$x_c = x + r_c\sin M \quad (27)$$

$$y_c = y + r_c\cos M \quad (28)$$

where

x, y = profile coordinates (Eq 23 and 24)

r_c = radius of cutter

The polar coordinates are

$$R_c = (x_c^2 + y_c^2)^{1/2} \quad (29)$$

$$\omega = 90° - (\Psi + \epsilon) \quad (30)$$

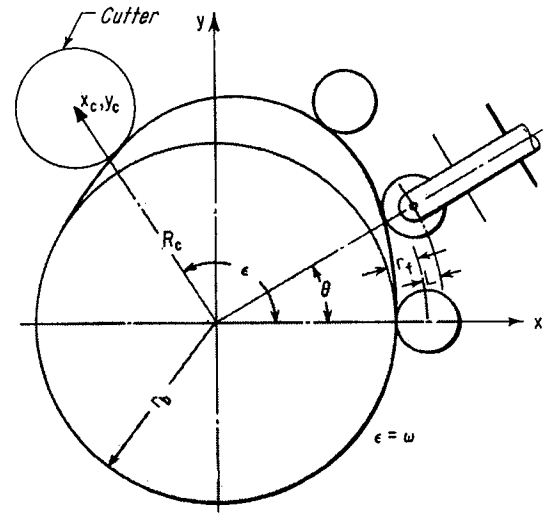
$$\epsilon = \tan^{-1}\frac{y_c}{x_c} \quad (31)$$

where

ϵ= angular displacement of the cutter with respect to the x axis, and with the cam stationary.

ω= angular displacement of the cutter center referenced to zero at the start of the cam profile rise, for cam specification purposes and convenience in machining.

The angles, ω, and the corresponding distances, R_c, are subject to adjustment to bring these values to even angles for convenience of machining. This will be illustrated later in the cam design example.



5 . . Radial cam with flat-face follower.

For offset swinging follower, the rectangular coordinates of the cutter are

$$x_c = x + r_c\sin M \quad (32)$$

$$y_c = y + r_c\cos M \quad (33)$$

and the polar cutter coordinates are

$$R_c = (x_c^2 + y_c^2)^{1/2} \quad (34)$$

$$\epsilon = \tan^{-1}\frac{y_c}{x_c} \quad (35)$$

Flat-face translating follower

The follower of this type of flat-face cam moves radially, Fig 5. The general equation of the family of lines forming the envelope is

$$y = mx + b$$

where

$m = \cos\theta$

L = lift of follower

$x = (r_b + L)\cos\theta$

$y = (r_b + L)\sin\theta$

$b = (r_b + L)/\sin\theta$

Hence

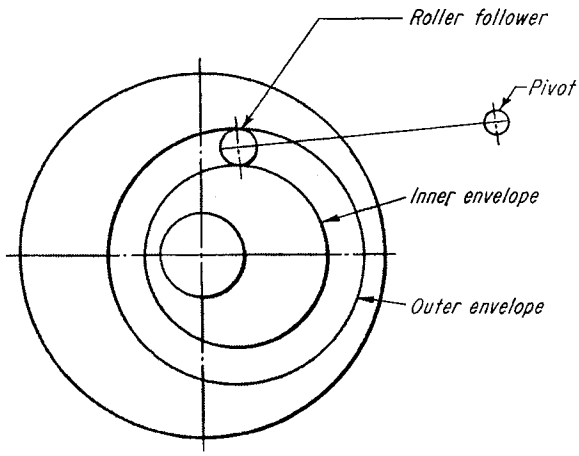
$$y = \frac{r_b + L - x\cos\theta}{\sin\theta} \quad (36)$$

Therefore

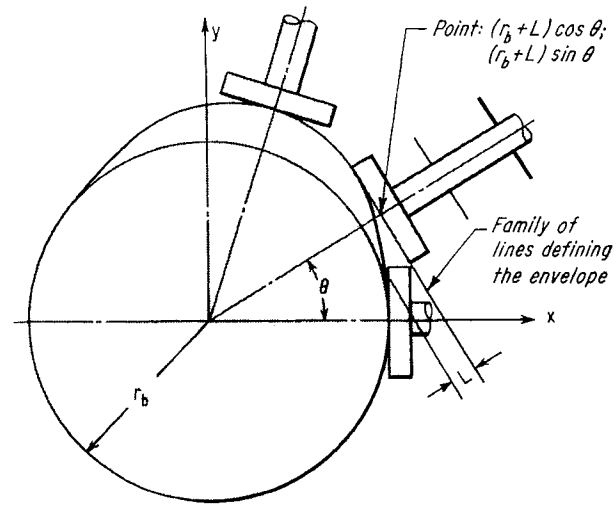
$$f(x,y,\theta) = y\sin\theta + x\cos\theta - (r_b + L) = 0 \quad (37)$$

and

$$\frac{df}{d\theta} = y\cos\theta - x\sin\theta - \frac{dL}{d\theta} = 0 \quad (38)$$



6 . . **Positive-action cam** with double envelope.



7 . . **Radial cam** with roller follower.

The profile coordinates are (by solving simultaneously Eq 37 and 38):

$$x = (r_b + L)\cos\theta - \frac{dL}{d\theta}\sin\theta \qquad (39)$$

$$y = (r_b + L)\sin\theta + \frac{dL}{d\theta}\cos\theta \qquad (40)$$

where L is usually given in terms of the cam angle θ (similar to ϕ for the swinging follower).

The rectangular coordinates are

$$x_c = x + r_c\cos\theta \qquad (41)$$

$$y_c = y + r_c\sin\theta \qquad (42)$$

Polar coordinates of profile points are obtained by squaring and adding Eq 39 and 40:

$$R^2 = x^2 + y^2$$

$$R = \left[(r_b + L)^2 + \left(\frac{dL}{d\theta}\right)^2\right]^{1/2} \qquad (43)$$

Cutter coordinates in polar form are obtained by squaring and adding Eq 41 and 42.

$$R_c = \left[(r_b + L + r_c)^2 + \left(\frac{dL}{d\theta}\right)^2\right]^{1/2} \qquad (44)$$

$$\omega = \tan^{-1}\frac{y_c}{x_c}$$

$$= \tan^{-1}\left[\frac{(r_b + L + r_c)\sin\theta + \frac{dL}{d\theta}\cos\theta}{(r_b + L + r_c)\cos\theta - \frac{dL}{d\theta}\sin\theta}\right]$$

ROLLER FOLLOWERS

In determining the profile of a roller-follower cam by envelope theory, two envelopes are mathematically possible—one the inner, profile envelope and the other an outer envelope. If a positive-action cam is to be constructed, Fig 6, both envelopes are applicable, since they constitute the slot in which the roller follower would be constrained to move to give the desired output motion.

The equations for three types of roller-follower cams are derived below.

Translating roller follower

The radial distance, H, to the center of the follower for this type of cam, Fig 7, is equal to:

$$H = r_b + r_f + L$$

where

r_f = radius of the follower roller

r_b = base circle radius

L = lift = $L(\theta)$

The general equation of the envelope is

$$(x - H\cos\theta)^2 + (y - H\sin\theta)^2 - r_f^2 = 0 \qquad (45)$$

The profile coordinates are (by applying $d/d\theta = 0$ and solving for y and x):

$$y = \frac{x\left[H\sin\theta - \frac{dL}{d\theta}\cos\theta\right] + H\frac{dL}{d\theta}}{H\cos\theta + \frac{dL}{d\theta}\sin\theta} \qquad (46)$$

and

$$x = H\cos\theta \pm \frac{r_f}{\left[1 + \left(\frac{H\sin\theta - \frac{dL}{d\theta}\cos\theta}{H\cos\theta + \frac{dL}{d\theta}\sin\theta}\right)^2\right]^{1/2}} \quad (47)$$

where

$$\frac{dL}{d\theta} = \frac{dH}{d\theta}$$

Here the plus-minus ambiguity may be resolved by examining $\theta = 0$ when $x = r_b$. At this point

$$H = r_b + r_f$$

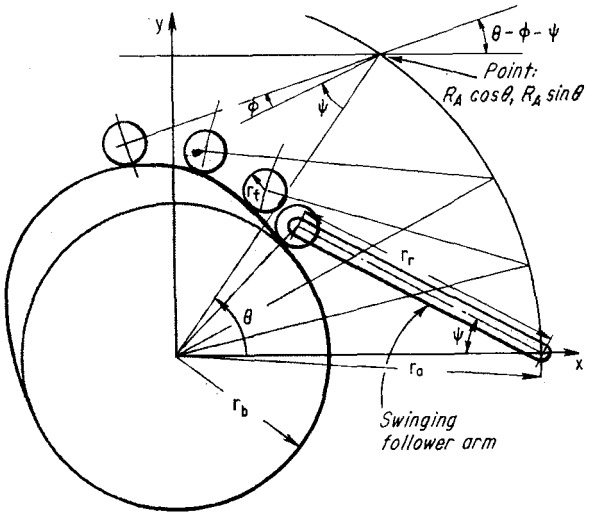
$$\frac{dH}{d\theta} = \frac{dL}{d\theta} = 0$$

and

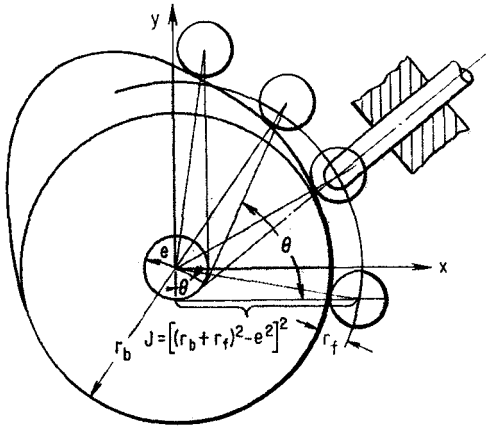
$$x = r_f + r_b \pm r_f$$

Only the negative sign is meaningful in the above equation; thus the negative sign in Eq 47 establishes the inner envelope, and the plus sign the outer envelope, which in this case is discarded.

The final equation for y can be computed by substituting Eq 47 into 46.



8 . . Swinging roller-follower cam.



9 . . Offset radial-roller cam.

Rectangular coordinates of the cutter are

$$x_c = x + \frac{r_c}{r_f}(H\cos\theta - x) \quad (48)$$

$$y_c = y + \frac{r_c}{r_f}(H\sin\theta - y) \quad (49)$$

Polar coordinates of the cutter are

$$R_c = (x_c^2 + y_c^2)^{1/2} \quad (50)$$

$$\epsilon = \tan^{-1}\frac{y_c}{x_c} \quad (51)$$

Swinging roller follower

This type of cam is illustrated in Fig 8. Angle ψ is equal to

$$\Psi = \cos^{-1}\frac{r_r^2 + r_a^2 - (r_b + r_f)^2}{2r_a r_r} \quad (52)$$

The general equation of the family is

$$[x - r_a\cos\theta + r_r\cos N]^2 + [y - r_a\sin\theta + r_r\sin N]^2 - r_f^2 = 0 \quad (53)$$

where

$$N = \theta - \phi - \Psi$$

The profile coordinates are (by the method outlined for the translating roller follower):

$$y = \frac{x\left[r_a\sin\theta - r_r\left(1 - \frac{d\phi}{d\theta}\right)\sin N\right]}{r_a\cos\theta - r_r\left(1 - \frac{d\phi}{d\theta}\right)\cos N} \quad (54)$$

and

$$x = r_a\cos\theta - r_r\cos N \pm \frac{r_f}{\left[1 + \left(\frac{r_a\sin\theta - r_r\left(1 - \frac{d\phi}{d\theta}\right)\sin N}{r_a\cos\theta - r_r\left(1 - \frac{d\phi}{d\theta}\right)\cos N}\right)^2\right]^{1/2}} \quad (55)$$

Referring to the $\pm$ sign, the negative sign gives the actual cam profile; the positive sign produces an outer envelope. The equation for y can be computed by substituting Eq 55 into 54.

The rectangular cutter coordinates are:

$$x_c = x + \frac{r_c}{r_f}(x_f - x) \quad (56)$$

$$y_c = y + \frac{r_c}{r_f}(y_f - y) \quad (57)$$

where x_f and y_f, the coordinates to the center of cutter, are equal to

$$x_f = r_a \cos\theta - r_r \cos N$$
$$y_f = r_a \sin\theta - r_r \sin N$$

The polar cutter coordinates are

$$R_c = (x_c^2 + y_c^2)^{1/2} \tag{58}$$

$$\epsilon = \tan^{-1}\frac{y_c}{x_c} \tag{59}$$

Translating offset roller follower

The roller follower of this type of cam, Fig 9, moves radially along a line that is offset from the cam center by a distance e.

The general equation of the envelope is

$$[x - e\sin\theta - (J + L)\cos\theta]^2 + [y + e\cos\theta - (J + L)\sin\theta]^2 - r_f^2 = 0 \tag{60}$$

where

$$J = [(r_b + r_f)^2 - e^2]^{1/2}$$

The profile coordinates are (by applying $d/d\theta = 0$, and solving for y and x):

$$y = \frac{x\left[(J+L)\sin\theta - \left(e+\frac{dL}{d\theta}\right)\cos\theta\right] + (J+L)\frac{dL}{d\theta}}{(J+L)\cos\theta + \left(e+\frac{dL}{d\theta}\right)\sin\theta} \tag{61}$$

$$x = e\sin\theta + (J + L)\cos\theta \pm \frac{r_f}{\left[1+\left(\frac{(J + L)\sin\theta - \left(e + \frac{dL}{d\theta}\right)\cos\theta}{(J + L)\cos\theta + \left(e + \frac{dL}{d\theta}\right)\sin\theta}\right)^2\right]^{1/2}} \tag{62}$$

Here again the negative sign of the plus-minus ambiguity is physically correct. The plus sign produces the outer envelope.

Final equation for y can be obtained by substituting Eq 62 into 61.

The rectangular cutter coordinates are

$$x_c = x + \frac{r_c}{r_f}(x_f - x) \tag{63}$$

$$y_c = y + \frac{r_c}{r_f}(y_f - y) \tag{64}$$

where

$$x_f = e\sin\theta + (J + L)\cos\theta$$
$$y_f = -e\cos\theta + (J + L)\sin\theta$$

The polar cutter coordinates are the same as Eqs 58 and 59.

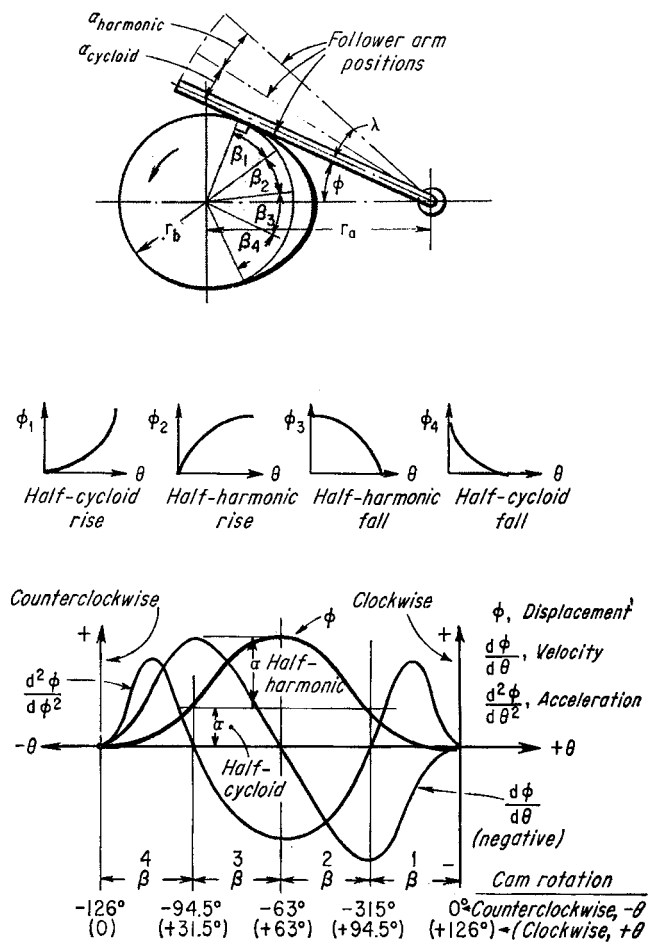
NUMERICAL EXAMPLE

The design specification

We have recently applied the cam equations to the design of a flat-faced swinging follower with face in line with the follower pivot. The follower oscillates through an output angle, λ, with a dwell-rise-fall-dwell motion.

The angular displacement of the follower arm is specified by portions of curves which can be expressed as mathematical functions of the angle of rotation of the cam. The specified angular motion of the arm consists of a half-cycloidal rise from the dwell, followed by half-harmonic rise and fall, and then by a half-cycloidal return to the dwell, as shown in Fig 10. Each region is 31.5 deg; the total cycle is completed in 126 deg.

Also included are the general shape of the follower velocity and acceleration curves, which result from: 1) the choice of curves, 2) the stipulation that the cam angle of rotation, β, for each curve segment be equal, and 3) the stipulation that the angular velocity at the matching points of the curves be the same for both curves. The cam is to rotate in the counterclockwise direction. It is to be specified by polar coordinates, R_c, ω, in 1-deg increments.



10 . . Cam design problem, illustrating cam layout, top, phase diagrams, center, and displacement diagram.

The equations of angular displacement for the four regions, or curve segments are

Rising—region 1 (half-cycloid)

$$\phi_1 = \alpha_C\left[\frac{\theta_1}{\beta_1} - \frac{1}{\pi}\sin\left(180^\circ \times \frac{\theta_1}{\beta_1}\right)\right]$$

Rising—region 2 (half-harmonic)

$$\phi_2 = \alpha_C + \alpha_H \sin\left(90^\circ \times \frac{\theta_2}{\beta_2}\right)$$

Falling—region 3 (half-harmonic)

$$\phi_3 = \alpha_C + \alpha_H \cos\left(90^\circ \times \frac{\theta_3}{\beta_3}\right)$$

Falling—region 4 (half-cycloid)

$$\phi_4 = \alpha_C\left[1 - \frac{\theta_4}{\beta_4} - \frac{1}{\pi}\sin\left(180^\circ \times \frac{\theta_4}{\beta_4}\right)\right]$$

where

λ = maximum displacement angle of follower arm = 2.820,997,8 deg

α_C = half-cycloidal angle of displacement of follower = 1.240,958,6 deg

α_H = half-harmonic angle of displacement of follower = 1.580,039,2 deg

$\beta = \theta$ maximum = maximum lift angle for a particular curve segment = −31.5 deg

θ = cam angle, degree of counterclockwise rotation, or in a negative direction

$\phi = f(\theta)$ = instantaneous angle of displacement of the follower

Subscripts:

1 = half-cycloidal, rising

2 = half-harmonic, rising

3 = half-harmonic, falling

4 = half-cycloidal, falling

Also given are:

r_a = 3.25 in.

r_b = 1.1758 in.

Ψ = 21.209,369,3 deg

For illustrative purposes, however, the computations are rounded to four decimal places.

Solution

Eq 23 and 24 will give the x and y coordinates of the profile. The derivative, $d\phi/d\theta$, is also the angular velocity of the follower.

The computations for locating the profile when θ = −40 deg are presented below. All angles are in degrees:

$$\phi_{-40^\circ} = \alpha_C + \alpha_H\left(\sin 90^\circ \times \frac{\theta_2}{\beta_2}\right)$$

$$= 1.2406 + 1.5800\left[\sin 90\left(\frac{-(40 - 31.5)}{-31.5}\right)\right]$$

$$= 1.8908 \text{ deg}$$

$$\frac{d\phi}{d\theta} = \frac{\pi}{2}\frac{\alpha_H}{\beta_2}\left(\cos 90\,\frac{\theta_2}{\beta_2}\right)$$

$$= -0.0718$$

$$M = \phi - \theta + \Psi = 1.8909 - (-40) + 21.2094$$

$$= 63.1002 \text{ deg}$$

From Eq 23:

$$x_{-40^\circ} = 3.25\left[\cos(-40) + \frac{\cos(63.1002 - 40)\cos(63.1002)}{(-0.718 - 1)}\right]$$

$$= 1.2278 \text{ in.}$$

Similarly, from Eq 24:

$$y = 0.3983 \text{ in.}$$

The cutter coordinates are obtained by means of Eq 27 through 31, and

$$x_c = x + r_c \sin M$$

$$= 1.2278 + 1.5 \sin 63.1002 = 2.5655 \text{ in.}$$

$$y_c = y + r_c \cos M$$

$$= 1.0769 \text{ in.}$$

$$R_c = (x_c^2 + y_c^2)^{1/2} = [2.5655^2 + 1.0769^2]^{1/2}$$

$$= 2.7823 \text{ in.}$$

$$\epsilon = \tan^{-1}\frac{1.0796}{2.5655} = 22.7713 \text{ deg}$$

$$\omega = 90 - (\Psi + \epsilon)$$

$$= 90 - (21.2094 + 22.7713) = 46.0193 \text{ deg}$$

Cams and Gears Team Up in Programmed Motion

Pawls and ratchets are eliminated in this design, which is adaptable to the smallest or largest requirements; it provides a multitude of outputs to choose from at low cost.

Theodore Simpson

A new and extremely versatile mechanism provides a programmed rotary output motion simply and inexpensively. It has been sought widely for filling, weighing, cutting, and drilling in automatic and vending machines.

The mechanism, which uses overlapping gears and cams (drawing below), is the brainchild of mechanical designer Theodore Simpson of Nashua, N. H.

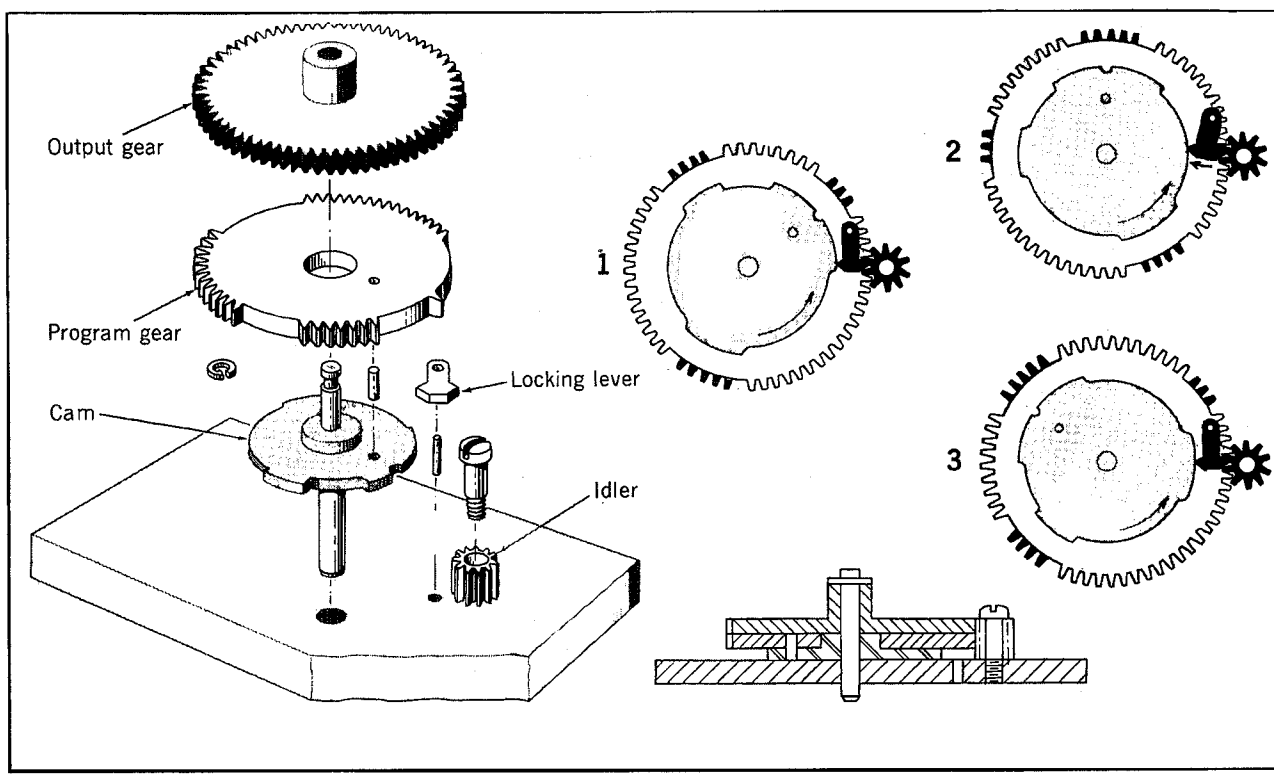
Based on a patented concept that could be transformed into a number of configurations, PRIM (Programmed Rotary Intermittent Motion), as the mechanism is called, satisfies the need for smaller devices for instrumentation without using spring pawls or ratchets.

It can be made small enough for a wristwatch or as large as required.

Versatile output. Simpson reports the following major advantages:

- Input and output motions are on a concentric axis.
- Any number of output motions of varied degrees of motion or dwell time per input revolution can be provided.
- Output motions and dwells are variable during several consecutive input revolutions.
- Multiple units can be assembled on a single shaft to provide an almost limitless series of output motions and dwells.
- The output can dwell, then snap around.

How it works. The basic model



Basic intermittent-motion mechanism, at left in drawings, goes through the rotation sequence as numbered above.

(drawing, below left) repeats the output pattern, which can be made complex, during every revolution of the input.

Cutouts around the periphery of the cam give the number of motions, degrees of motion, and dwell times desired. Tooth sectors in the program gear match the cam cutouts.

Simpson designed the locking lever so one edge follows the cam and the other edge engages or disengages, locking or unlocking the idler gear and output. Both program gear and cam are lined up, tooth segments to cam cutouts, and fixed to the input shaft. The output gear rotates freely on the same shaft, and an idler gear meshes with both output gear and segments of the program gear.

As the input shaft rotates, the teeth of the program gear engage the idler. Simultaneously, the cam releases the locking lever and allows the idler to rotate freely, thus driving the output gear.

Reaching a dwell portion, the teeth of the program gear disengage from the idler, the cam kicks in the lever to lock the idler, and the output gear stops until the next program-gear segment engages the idler.

Dwell time is determined by the space between the gear segments. The number of output revolutions does not have to be the same as the number of input revolutions. An idler of a different size would not affect the output, but a cluster idler with a matching output gear can increase or decrease the degrees of motion to meet design needs.

For example, a step-down cluster with output gear to match could reduce motions to fractions of a degree, or a step-up cluster with matching output gear could increase motions to several complete output revolutions.

Snap action. A second cam and a spring are used in the snap-action version (drawing below). Here, the cams have identical cutouts.

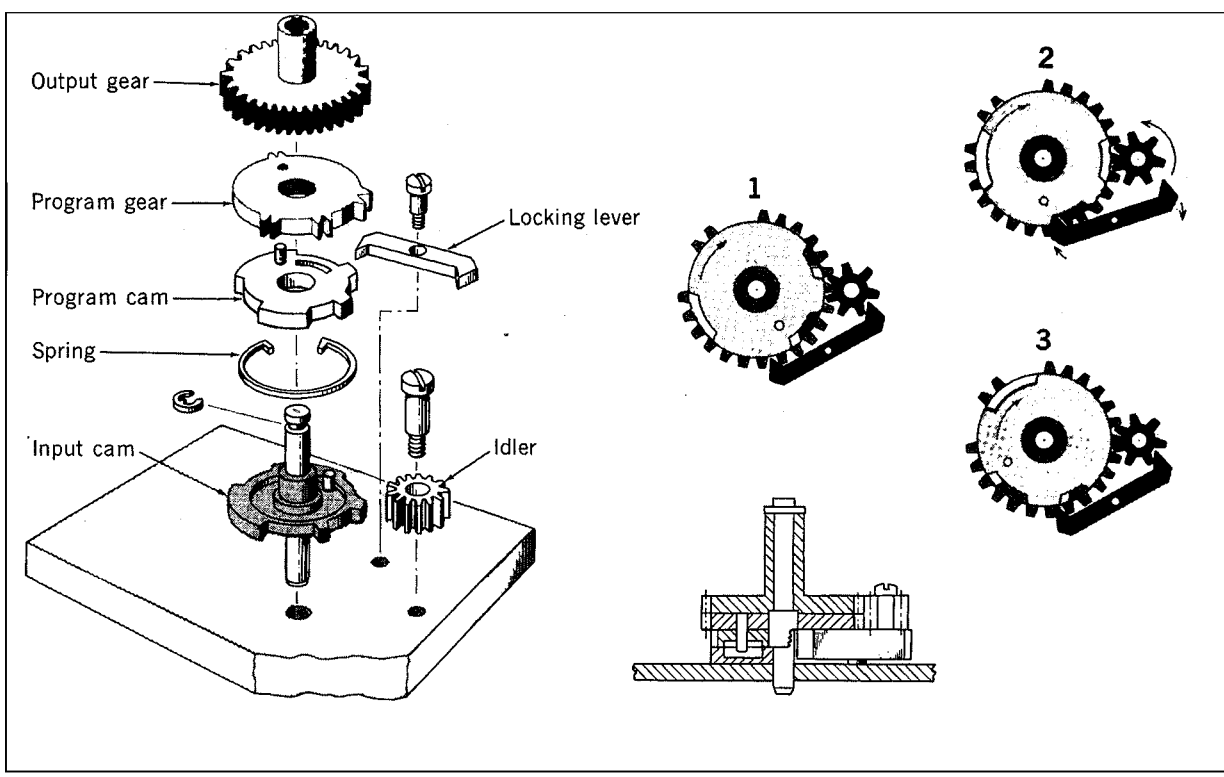
One cam is fixed to the input and the other is lined up with and fixed to the program gear. Each cam has a pin in the proper position to retain a spring; the pin of the input cam extends through a slot in the program gear cam that serves the function of a stop pin.

Both cams rotate with the input shaft until a tooth of the program gear engages the idler, which is locked and stops the gear. At this point, the program cam is in position to release the lock, but misalignment of the peripheral cutouts prevents it from doing so.

As the input cam continues to rotate, it increases the torque on the spring until both cam cutouts line up. This positioning unlocks the idler and output, and the built-up spring torque is suddenly released. It spins the program gear with a snap as far as the stop pin allows; this action spins the output.

Although both cams are required to release the locking lever and output, the program cam alone will relock the output—a feature of convenience and efficient use.

After snap action is complete and the output is relocked, the program gear and cam continue to rotate with the input cam and shaft until they are stopped again when a succeeding tooth of the segmented program gear engages the idler and starts the cycle over again.



Snap-action version, with a spring and with a second cam fixed to the program gear, works as shown in numbered sequence.

Minimum Cam Size

Whether for high-accuracy computers or commercial screw machines–here's your starting point for any can design problem.

Preben W. Jensen

THE best way to design a cam is first to select a maximum pressure angle—usually 30 deg for translating followers and 45 deg for swinging followers—then lay out the cam profile to meet the other design requirements. This approach will ensure a minimum cam size.

But there are at least six types of profile curves in wide use today—constant-velocity, parabolic, simple harmonic, cycloidal, 3-4-5 polynomial, and modified trapezoidal—and to design the cam to stay within a given pressure angle for any given curve is a time-consuming process. Add to this the fact that the type of follower employed also influences the design, and you come up with a rather difficult design problem.

You can avoid all tedious work by turning to the unique design charts presented here (Fig 5 to 10). These charts are based on a construction method (Fig 1 to 4) developed in Germany by Karl Flocke back in 1931 and published by the German VDI as Research Report 345. Flocke's method is practically unknown in this country—it does not appear in any published work. It is repeated here because it is a general method applicable to any type of cam curve or combination of curves. With it you can quickly determine the minimum cam size and the amount of offset that a follower needs—but results may not be accurate in that the points of max pressure angle must be estimated.

The design charts, on the other hand, are applicable only to the six types of curves listed above. But they are much quicker to use and provide more accurate results.

Also included in this article are

Symbols

e = offset (eccentricity) of cam-follower centerline with camshaft centerline, in.

R_b = base radius of cam, in.

R_f = roller radius, in.

R_{min} = minimum radius to pitch curve, in.; $R_{min} = R_b + R_f$

R_{max} = maximum radius to pitch curve, in.

y = linear displacement of follower, in.

y_{max} = prescribed maximum cam stroke, in.

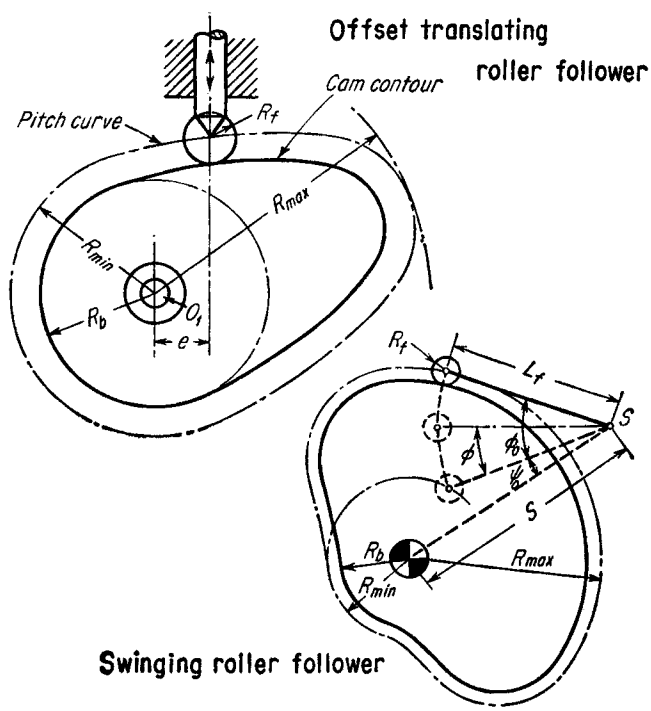
L_f = length of swinging follower arm, in.

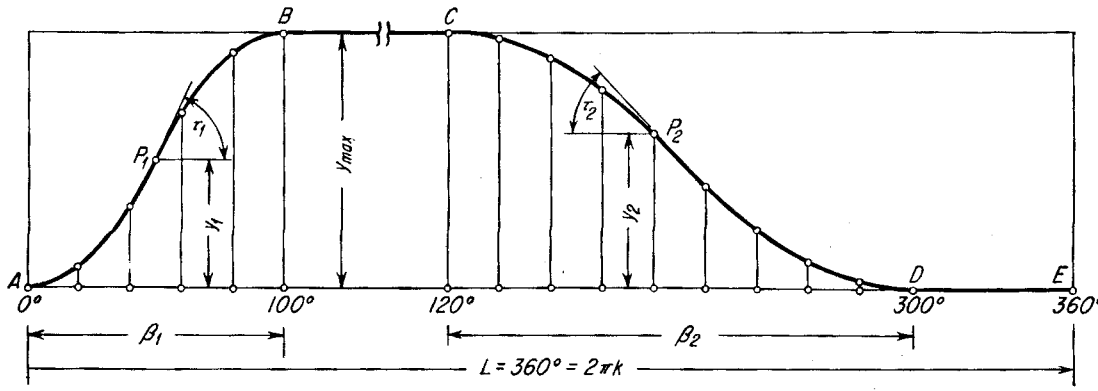
α = pressure angle, deg—the angle between the cam-follower centerline and the normal to the cam surface at the point of roller contact

β = cam angle rotation, deg

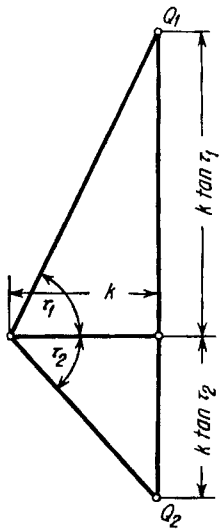
ϕ_o = angle of oscillation of swinging follower, deg

τ = slope of cam diagram, deg

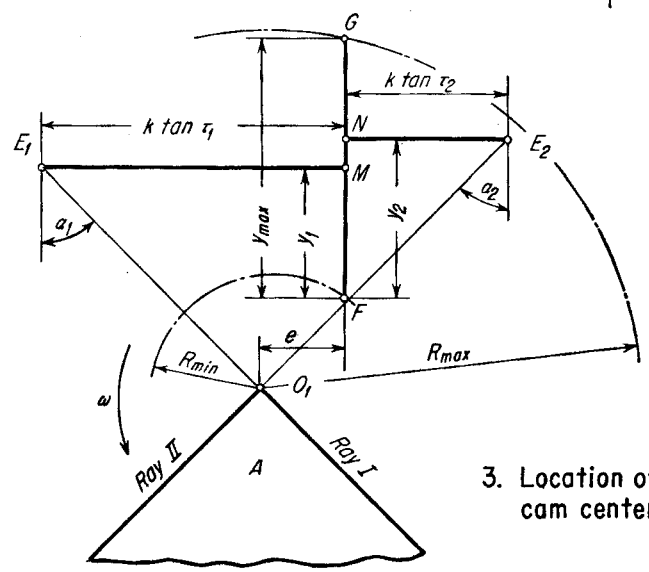




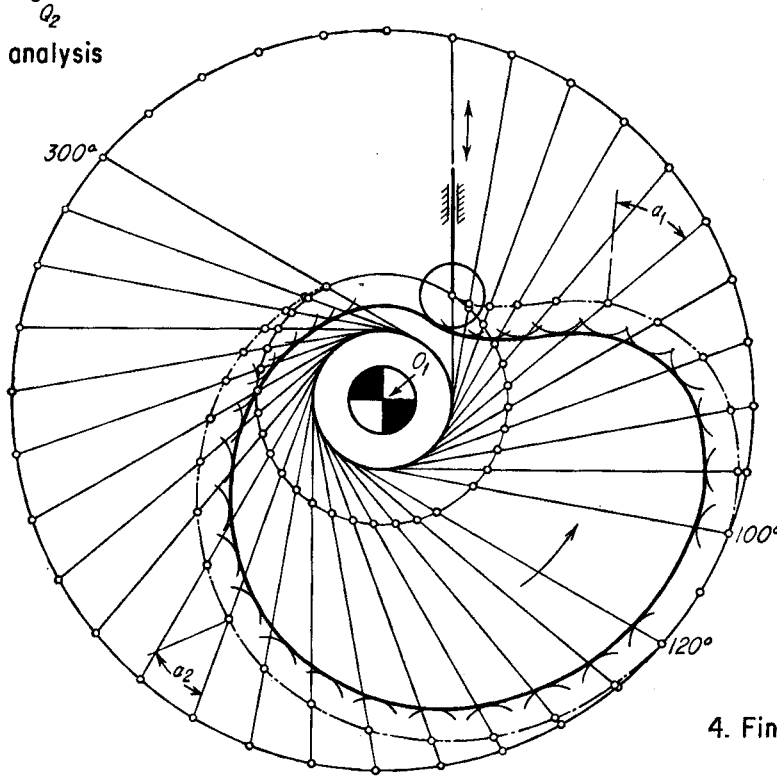
1. Cam diagram



2. Slope analysis



3. Location of cam center



4. Final cam shape

eight mechanisms for reducing the pressure angle when the maximum permissible pressure angle must be exceeded for one reason or another.

Why the emphasis on pressure angle?

Pressure angle is simply the angle between the direction where the follower wants to go and the direction where the cam wants to push it. Pressure angles should be kept small to reduce side thrusts on the follower. But small pressure angles increase cam size which in turn:

- Increases the size of the machine.
- Increases the number of precision points and cam material in manufacturing.
- Increases the circumferential speed of the cam which leads to unnecessary vibrations in the machine.
- Increases the cam inertia which slows up starting and stopping times.

Translating followers

Flocke's method for finding the minimum cam size—in other words e, R_{min} and R_{max} (see list of symbols)—is as follows:

In Fig 1

1. Lay out the cam diagram (time-displacement diagram) as the problem requires. Type of curve to be employed—parabolic, harmonic, etc—depends upon the requirements. Cam rise is during portion of curve AB; cam return, during CD.

2. Choose points of maximum slope during rise and return (points P_1 and P_2). The maximum pressure angle will occur near, or sometimes at, these points.

3. Measure slope angles τ_1 and τ_2.

4. Measure the length, L, in inches corresponding to 360 deg.

5. Calculate k:

$$k = \frac{L}{2\pi}$$

In Fig 2

1. Lay out k and angles τ_1, τ_2. This locates points Q_1 and Q_2.

2. Measure $k(\tan \tau_1)$ and $k(\tan \tau_2)$.

In Fig 3

1. Lay out vertical line, FG, equal to total displacement, y_{max}.

2. Lay out from point F, the displacements y_1 and y_2 (at points P_1 and P_2). This locates points M and N.

3. Lay out $k(\tan \tau_1)$ to left of M to obtain point E_1. Similarly k tan τ_2 to right from N locates E_2 (for CCW rotation of cam).

4. At points E_1 and E_2 locate the desired (usually maximum permissible) pressure angles of points P_1 and P_2. These angles are designated as α_1 and α_2.

5. The lines define the limits of an area A. Any cam shaft center chosen within this area will result in pressure angles at points P_1 and P_2 which will be equal to or less than the prescribed angles α_1 and α_2. If the cam shaft center is chosen anywhere along Ray I, the pressure angle at E_1 will be exactly α_1 (and similarly along Ray II for α_2). Thus, if O_1, the intersection of these two rays, is chosen as the cam center, the layout will provide the desired pressure angles for both rise and return.

6. The construction results in an offset roller follower whose eccentricity, e, is measured directly on the drawing. Radii R_{min} and R_{max} are also measured directly on the drawing. The actual cam shape is drawn to scale in Fig 4.

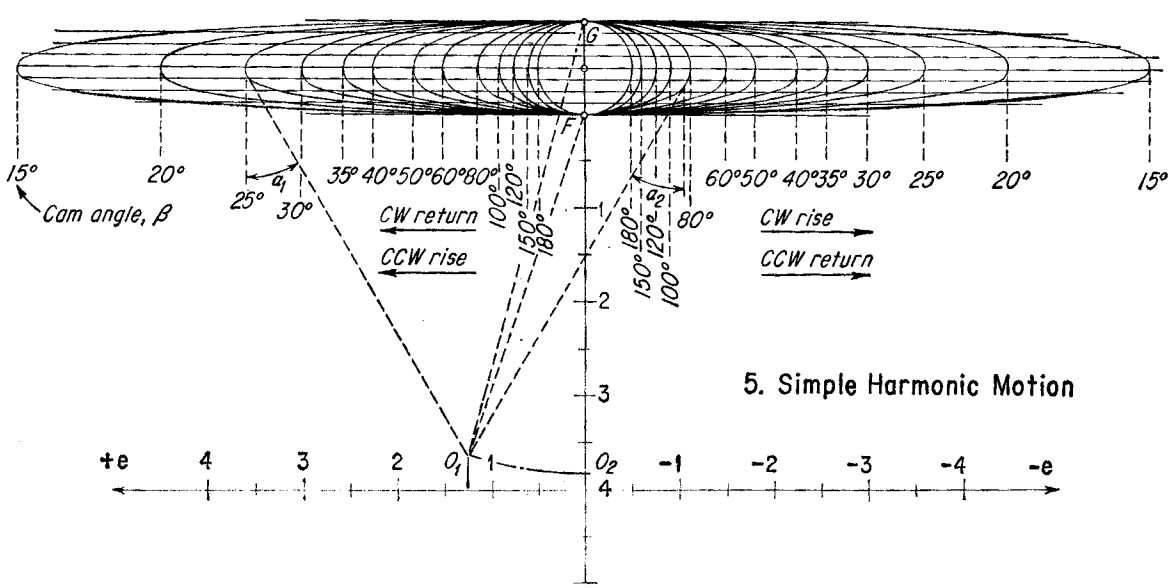
Design charts

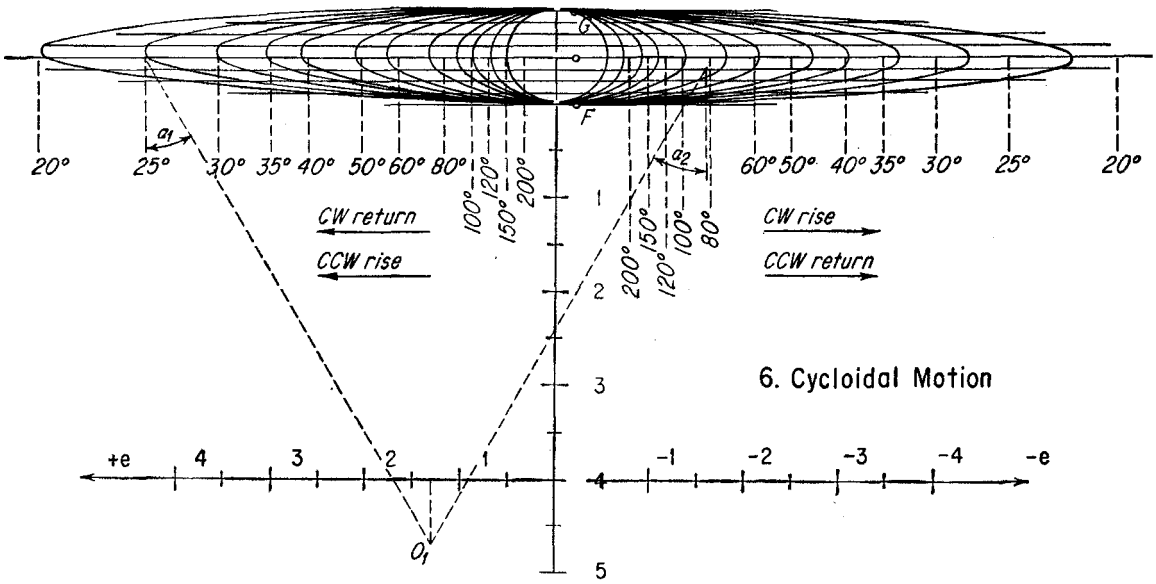
The above procedure, however, does not ensure that the pressure angle is not exceeded at some other point. *Only for some cases of parabolic motion will the maximum pressure angle occur at the point of maximum slope.* Thus the same procedure has to be repeated for numerous points during the rise and return motions. The six charts (Fig 5 to 10) developed by the author avoid the need for repetitive construction. Also, for cases where the cam size has already been chosen, the charts provide the maximum pressure angle during rise and return motions.

The scale of all the charts assumes that the stroke is equal to one unit. Hence, if $y_{max} = 1$ in. then the scales can be read off directly in inches.

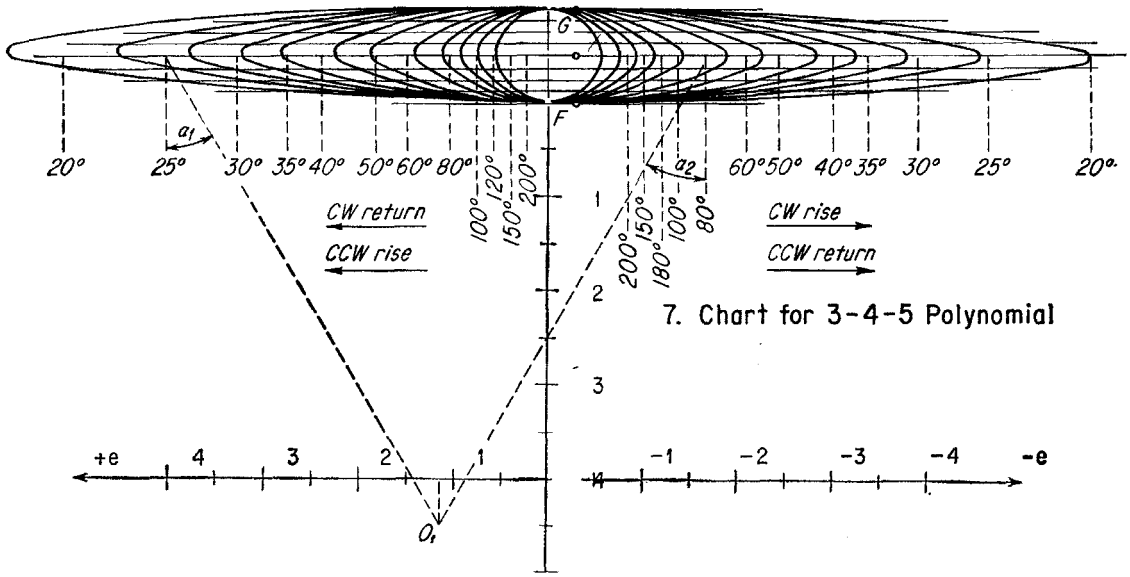
Design problem

All charts, Fig 5 to 10, show construction for the case where cam rotation during cam rise and fall respectively is $\beta_1 = 25$ deg and $\beta_2 = 80$ deg; total stroke, $y_{max} = 2$ in; max





6. Cycloidal Motion



7. Chart for 3-4-5 Polynomial

pressure angle during rise, $\alpha_1 = 30$ deg; during return, $\alpha_2 = 30$ deg. The cam rotates counterclockwise (CCW). Assume simple harmonic motion (Fig 5).

Construction

1. Because rotation is CCW, go to the left of center for the rise stroke, and to the right for the return stroke, as noted on the chart. Thus go to the $\beta = 25$ deg curve and layout angle $\alpha_1 = 30$ deg *tangent* to the curve. Lay out α_2 tangent to the $\beta = 80$ deg curve.

2. The point where the two lines intersect locates the cam shaft center O_1.

3. Read down to the e scale to obtain the required eccentricity. Hence $e = (1.23)(2) = 2.46$ in. (multiply by 2 because $y_{max} = 2$ in.).

4. Distance O_1F is R_{min}. To obtain its scale value, swing an arc from F to locate O_2. Hence $R_{min} = (3.85)(2) = 7.7$ in.

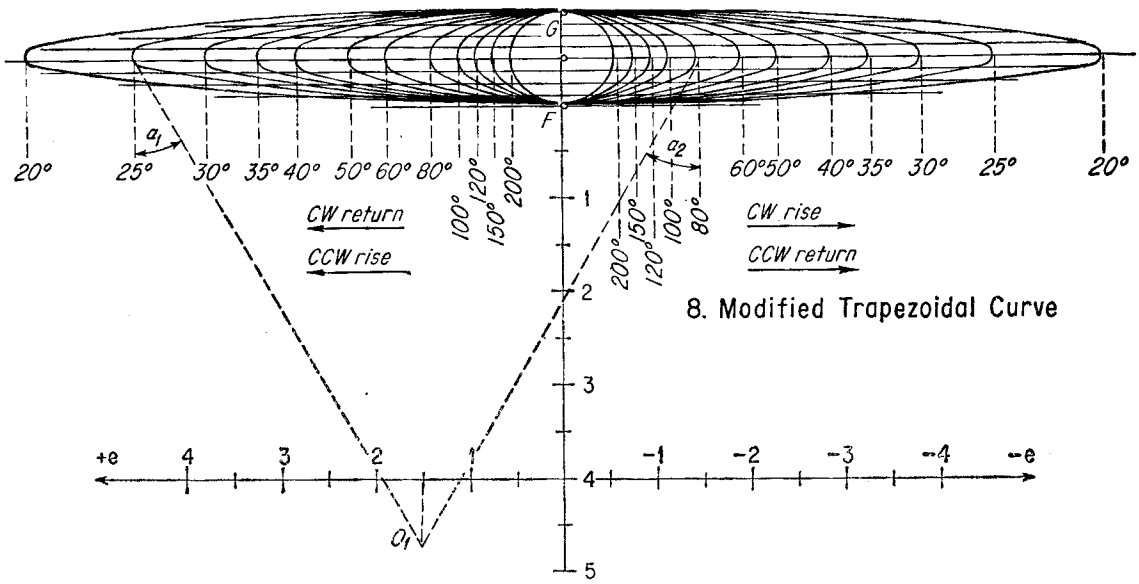
5. Distance $O_1G = R_{max} = (4.83)(2) = 9.66$ in.

All dimensions required to construct the minimum cam size are now known. You can also determine what part of the stroke the maximum pressure angle will occur at by noting the points of tangency of the α_1 and α_2 lines to the 25-deg and 80-deg curves. Extend these points horizontally to the FG line. Thus the max pressure angle occurs $\frac{7}{16}$ of the stroke upward during rise, and $\frac{11}{16}$ of the stroke downward during return.

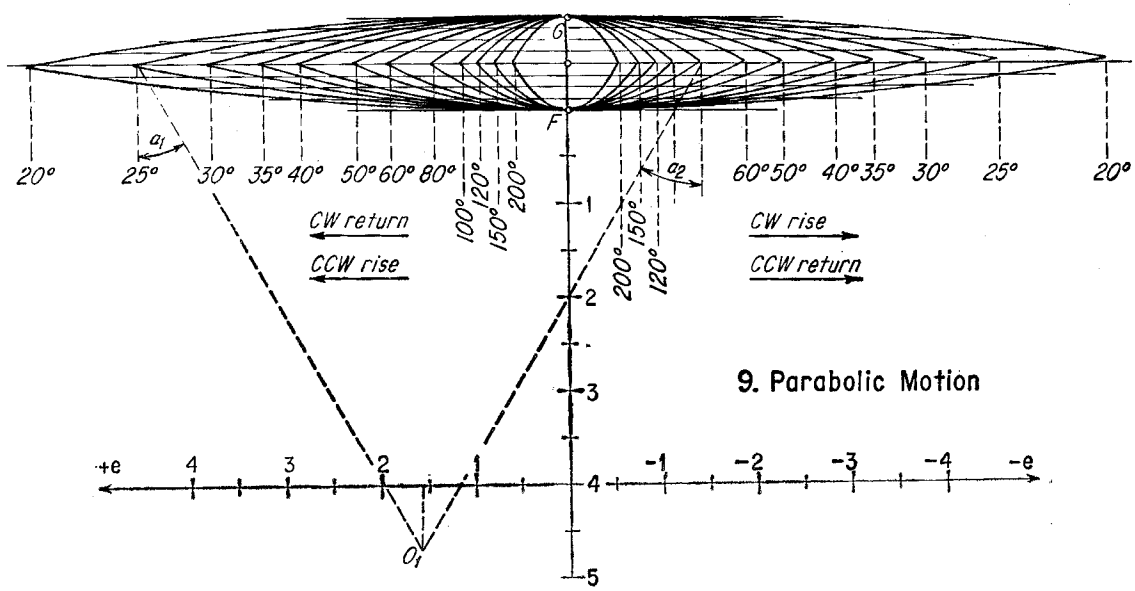
If you want to know the pressure angle, say at a point one quarter of the stroke during rise, go upward one quarter of the distance from F to G, then to the left to intersect the 25 deg curve. Connect this point of intersection to O_1. The angle that this new line makes with the vertical will be the requested pressure angle.

For parabolic cams

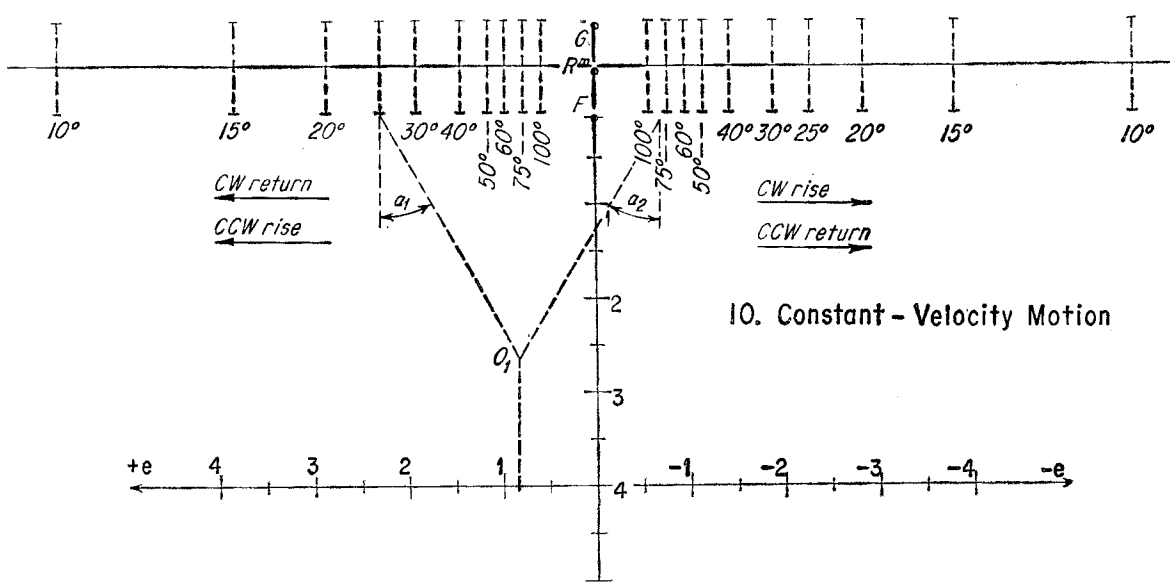
The procedure is slightly different here (Fig 9). The elongated curves are pointed at the ends. Thus the lines for pressure angles α_1 and α_2 are not tangent to the curve for the numerical



8. Modified Trapezoidal Curve



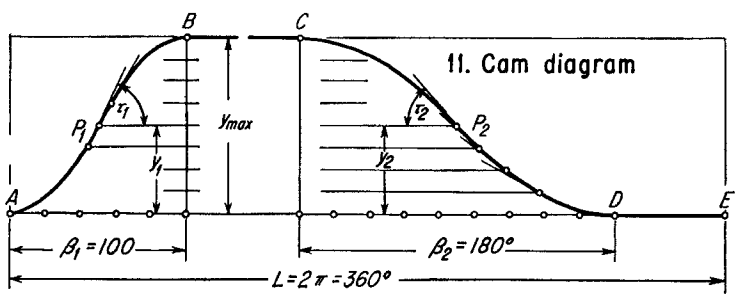
9. Parabolic Motion



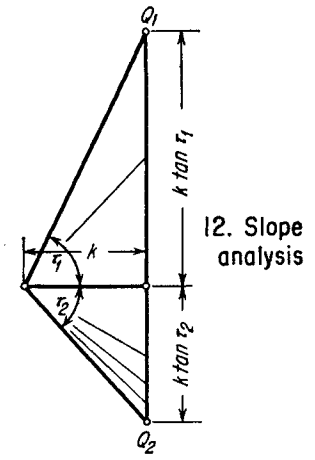
10. Constant - Velocity Motion

Comparison of Cam Sizes

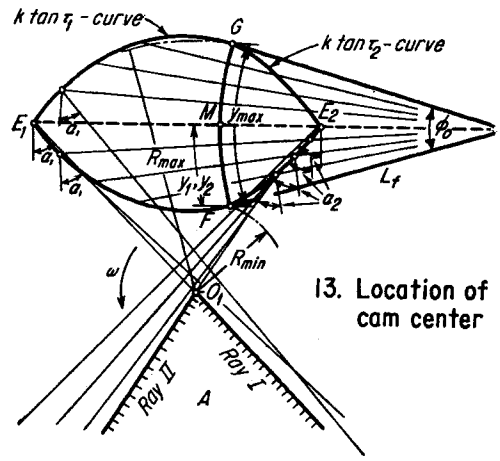
Type of cam	Max radius, R_{max}	Eccentricity, e
Constant velocity	3.65	0.8
Parabolic motion	5.90	1.58
Simple harmonic motion	4.80	1.24
Cycloidal motion	5.90	1.58
3-4-5 Polynomial	5.65	1.48
Modified trapezoidal	5.90	1.57
Double harmonic motion	5.85	1.60



11. Cam diagram



12. Slope analysis



13. Location of cam center

conditions given (but in some cases the lines may be).

For constant velocity cams

The elongated curves for this type of motion become vertical lines (Fig 10). Use the lower points of these lines for laying out α_1 and α_2, as shown by the dashed lines.

Comparison of cam sizes

A comparison of the required cam sizes for the six types of cam contours is given at the top of this page. Note that the constant-velocity curve requires the smallest cam size.

Swinging followers

Cams with swinging followers require a construction technique similar to the Flocke method described previously.

Assume that a cam diagram is given (Fig 11). Also known are the length of follower arm, L_f, and the angle of arm oscillation during rise and fall, ϕ_o. The length of the circular arc through which the roller follower swings must be equal to y_{max} in Fig 11. (See p. 69 for an illustration of a swinging follower cam.)

The construction technique, illustrated in Fig 11, 12 and 13, is as follows:

1. Divide the ordinate of the cam diagram into equal parts (8 in this case).

2. Select points along the divisions and find the slope angles at the points. The procedure is shown only for points P_1 and P_2, but it should be repeated for other points.

3. Calculate $k = L/(2\pi)$. In Fig 12, lay off k and angles τ_1 and τ_2. Obtain $k(\tan \tau_1)$ and $k(\tan \tau_2)$.

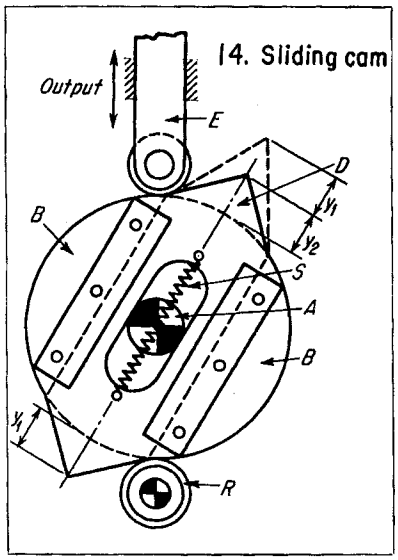
4. In Fig 13 lay off L_f (from S to F) and divide ϕ_o into 8 equal parts.

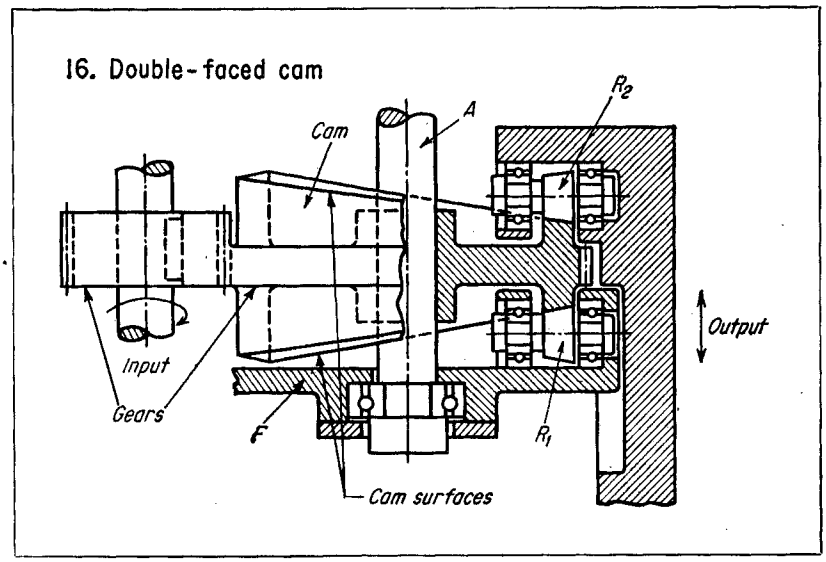
5. Lay out y_1 and y_2 as shown (in this case y_1 and y_2 are equal).

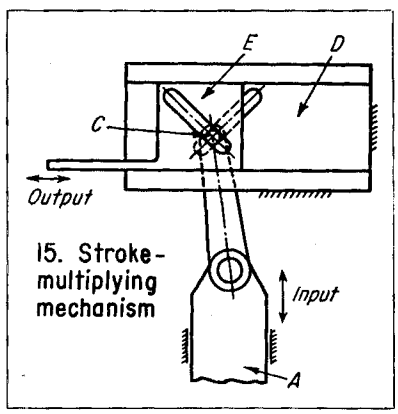
6. If cam rotation is away from pivot point S (counterclockwise in this case) lay off $ME_1 = k \tan \tau_1$ to the left of point M, and $ME_2 = k \tan \tau_2$ to the right of point M. (Reverse directions for clockwise rotation of cam.)

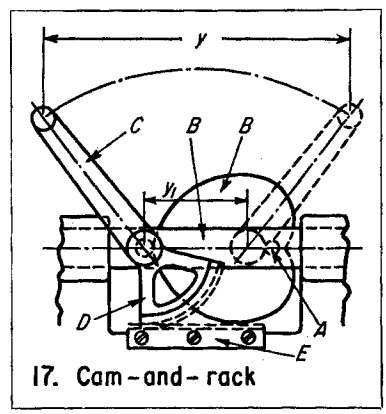
8. Lay out α_1 and α_2 at E_1 and E_2. Repeat procedure for other points as shown. Now choose the lowest line from both ends to obtain an area, A, which is the farthest area possible from F. This results in Ray I and Ray II. If a cam shaft center is chosen anywhere within this area, the maximum pressure angle will not be exceeded, either during rise or return.

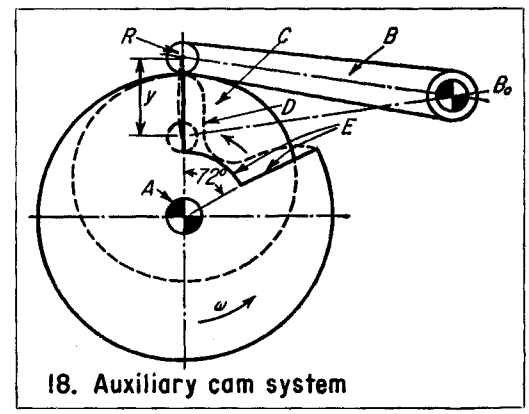
9. If O_1 is chosen, the maximum pressure angles during rise will occur at the middle of the stroke because Ray I is determined from E_1, which in turn corresponds to the middle of the stroke. Note that the maximum pressure angle for the return stroke will occur when the follower moves back ⅞ of the stroke because Ray II originates from a point ⅞ of angle ϕ_o measured downward from the top.











When the pressure angles are too high to satisfy the design requirements, and it is undesirable to enlarge the cam size, then certain devices can be employed to reduce the pressure angles:

Sliding cam, Fig 14—This device is used on a wire-forming machine. Cam *D* has a rather pointed shape because of the special motion required for twisting wires. The machine operates at slow speeds, but the principle employed here is also applicable to high-speed cams.

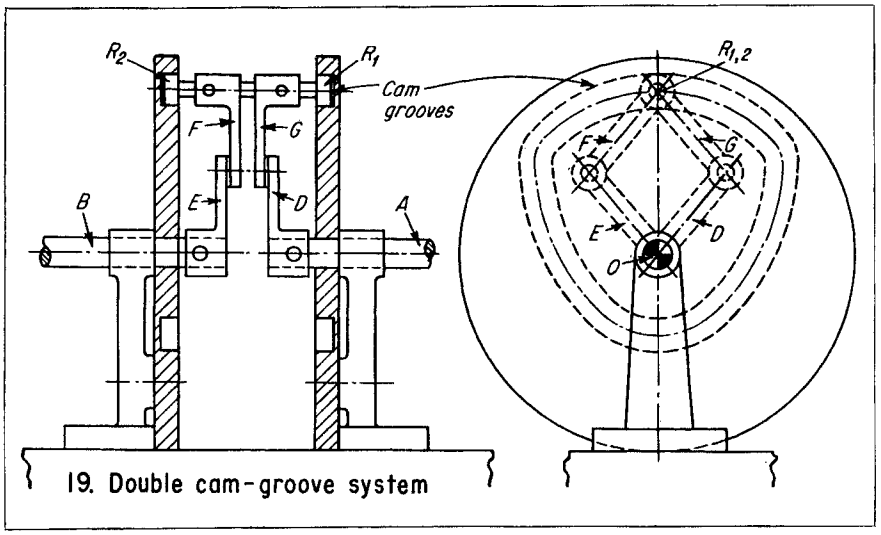
The original stroke desired is $(y_1 + y_2)$ but this results in a large pressure angle. The stroke therefore is reduced to y_2 on one side of the cam, and a rise of y_1 is added to the other side. Flanges *B* are attached to cam shaft *A*. Cam *D*, a rectangle with the two cam ends (shaded), is shifted upward as it cams off stationary roller *R*, during which the cam follower *E* is being cammed upward by the other end of cam *D*.

Stroke multiplying mechanism, Fig 15—This device is employed in power presses. The opposing slots, one in a fixed member *D* and the second in the movable slide *E*, multiply the motion of the input slide *A* driven by the cam. As *A* moves upward, *E* moves rapidly to the right.

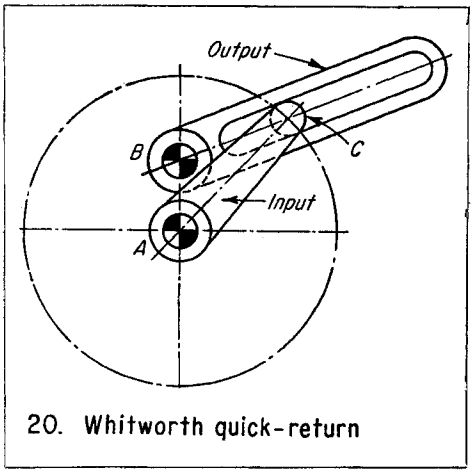
Double-faced cam, Fig 16 — This device doubles the stroke, hence reduces the pressure angles to one-half their original values. Roller R_1 is stationary. When the cam rotates, its bottom surface lifts itself on R_1, while its top surface adds an additional motion to the movable roller R_2. The output is driven linearly by roller R_2 and thus is approximately the sum of the rise of both surfaces.

Cam-and-rack, Fig 17—This device increases the throw of a lever. Cam *B* rotates around *A*. The roller follower travels at distances y_1, during which time gear segment *D* rolls on rack *E*. Thus the output stroke of lever *C* is the sum of transmission and rotation giving the magnified stroke *y*.

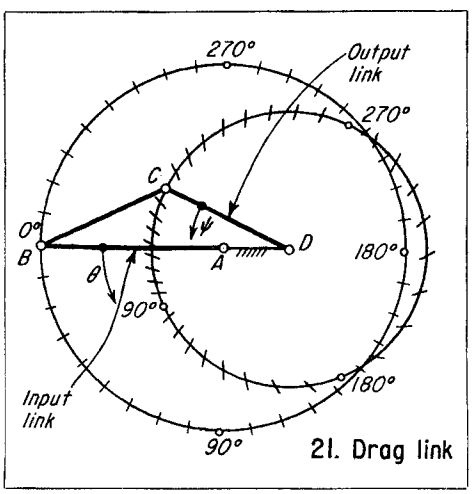
Cut-out cam, Fig 18—A rapid rise and fall within 72 deg was desired. This originally called for the cam contour, *D*, but produced severe pressure angles. The condition was improved by providing an additional cam *C* which also rotates around the cam center *A*, but at five times the speed of cam *D* because of a 5:1 gearing arrangement (not shown). The original cam is now completely cut away for the 72 deg (see surfaces *E*). The desired motion, expanded over 360 deg (since 72 x 5 = 360), is now designed into cam *C*. This results in the same pressure angle as would occur if the original cam rise occurred over 360 deg instead of 72 deg.



19. Double cam-groove system

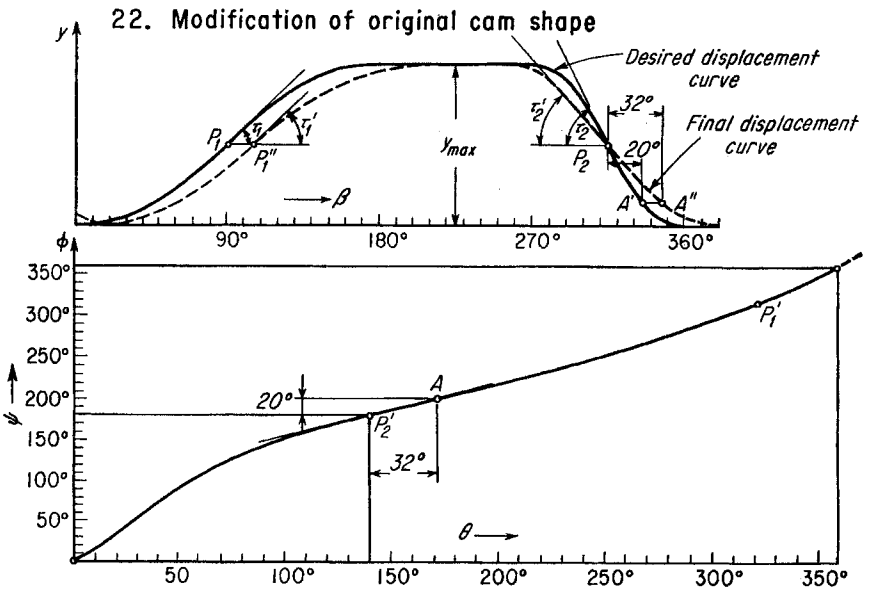


20. Whitworth quick-return



21. Drag link

22. Modification of original cam shape



Double-cam mechanism, Fig 19—If you were to increase the cam speed at the point of high-pressure angles, and change the contour accordingly, the pressure angle would be reduced. The device in Fig 19 employs two cam grooves to change the input speed A to the desired varying-speed output in shaft B. Shaft B then becomes the cam shaft to drive the actual cam (not shown). If the cam grooves are circular about point O then the output will be a constant velocity. Distance OR therefore is varied to provide the desired variation in output.

Whitworth quick return mechanism, Fig 20—This is a simpler way of imparting a varying motion to the output shaft B. However, the axes, A and B, are not colinear.

Drag link, Fig 21—This is another simple device for varying the output motion of shaft D. Shaft A rotates with uniform speed. The construction in Fig 22 shows how to modify the original cam shape to take full advantage of the varying input motion provided by shaft D. The construction steps are as follows (the desired displacement curve is given at the top of Fig 22, with the maximum pressure angle designated as τ_2):

1. Plot the input vs output diagram (θ vs ψ) for the linkage illustrated in Fig 21.
2. Find the point with the smallest slope, P_2.
3. Pick any point A on the tangent to P'_2 and measure the corresponding angles to P'_2 (32 deg and 20 deg).
4. Go 20 deg to the right of P_2 in the cam diagram to locate A'. Also locate A by going 32 deg to the right of P_2 as shown. Point A' is on the final cam shape. Repeat this procedure with more points until you obtain the final curve. The pressure angle at P_2 is thus reduced from τ_2 to τ_2'.

Spherical Cams: Linking Up Shafts

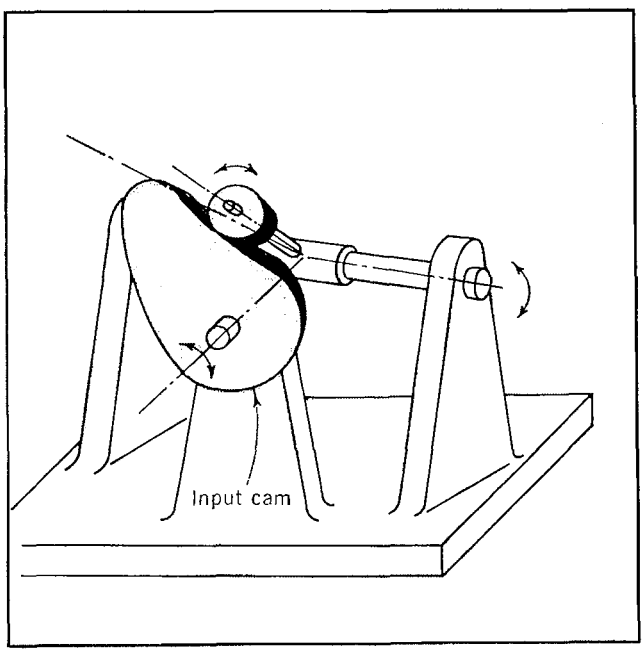
European design is widely used abroad but little-known in the U.S. Now a German engineering professor is telling the story in this country, stirring much interest.

Anthony Hannavy

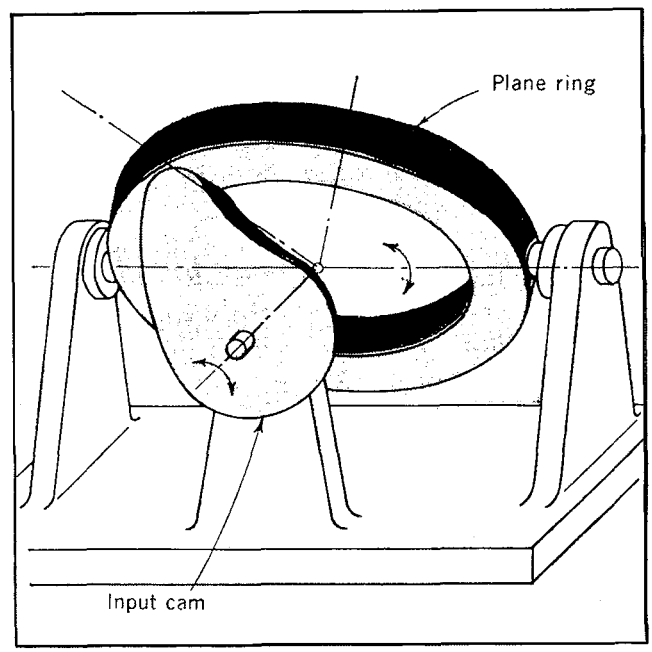
Problem: to transmit motion between two shafts in a machine when, because of space limitations, the shaft axes may intersect each other. One answer is to use a spherical-cam mechanism, unfamiliar to most American designers but used in Europe to provide many types of motion in agricultural, textile, and printing machinery.

Recently, Prof. W. Meyer zur Cappellen of the Institute of Technology, Aachen, Germany, visited the U. S. to show designers how spherical-cam mechanisms work and how to design and make them. He and his assistant kinematician at Aachen, Dr. G. Dittrich, are in the midst of experiments with complex spherical-cam shapes and with the problems of manufacturing them.

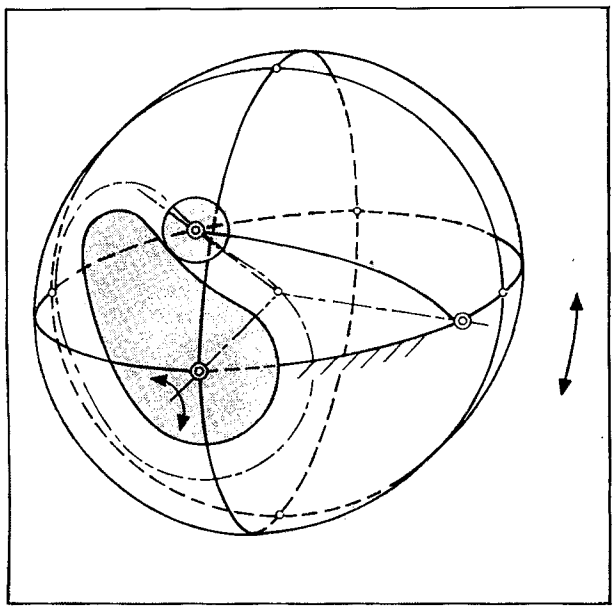
Fundamentals. Key elements of spherical-cam mechanism (above Fig. 1) can be considered as being posi-



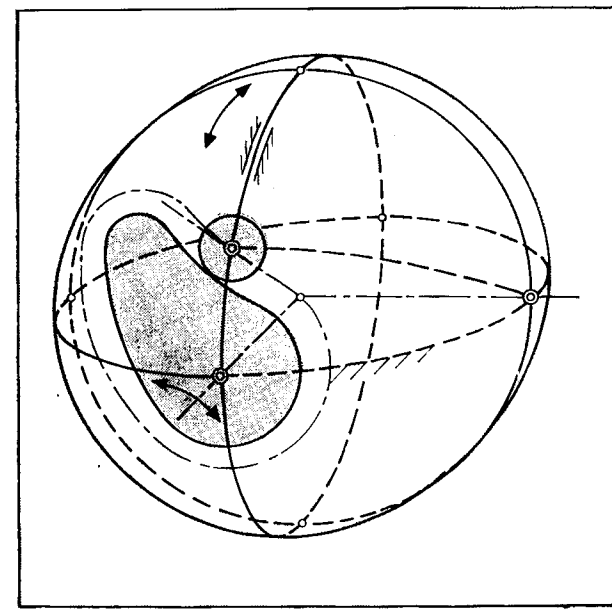
3 Spherical mechanism with radial follower



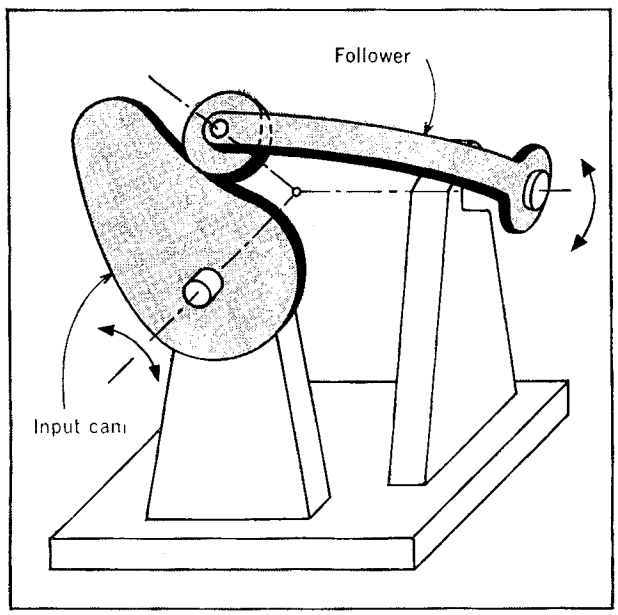
4 Cam mechanism with flat-faced follower



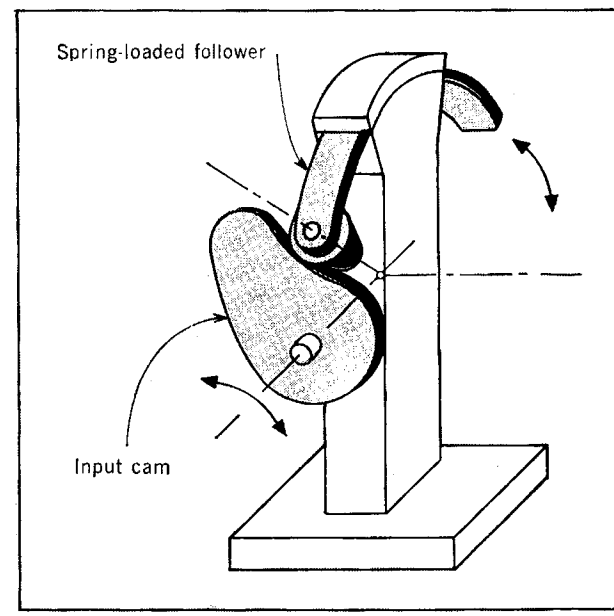
Radial roller follower shown on a sphere



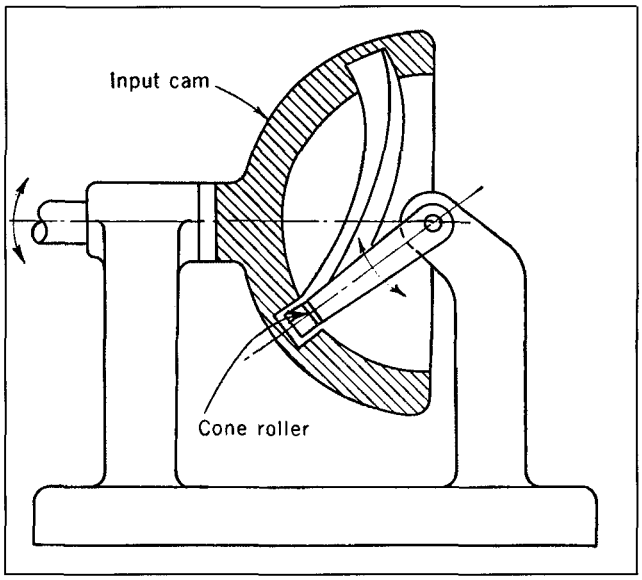
Mechanism with radial roller follower shown on a sphere



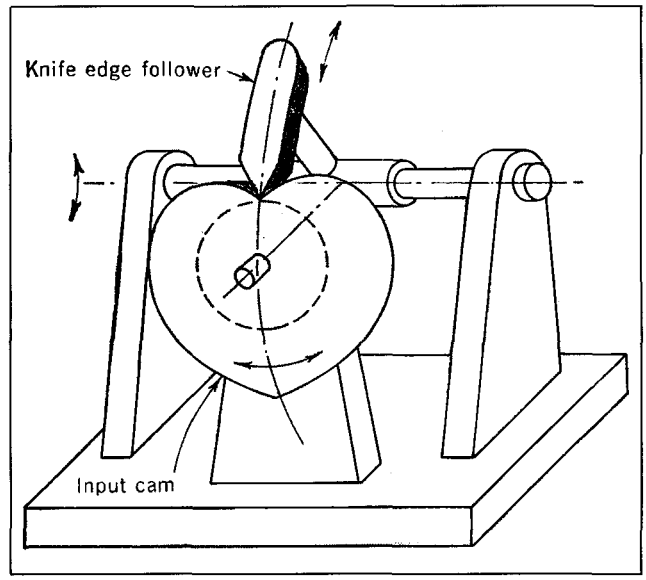
1 Spherical cam mechanism with radial follower



2 Cam mechanism with rocking roller follower



5 Hollow-sphere cam mechanism



6 Mechanism with Archimedean spiral; knife-edge follower

tioned on a sphere. The center of this sphere is the point where the axes of rotation of the input and follower cams intersect.

In a typical configuration in an application (Fig. 1), the input and follower cams are shown with depth added to give them a conical roller surface. The roller is guided along the conical surface of the input cam by a rocker, or follower.

A schematic view of a spherical-cam mechanism (above Fig. 2) shows how the follower will rise and fall along a linear axis. In the same type of design (Fig. 2), the follower is spring-loaded. The designer can also use a rocking roller follower (Fig. 3) that oscillates about an axis that, in turn, intersects with another shaft.

These spherical-cam mechanisms using a cone roller have the same output motion characteristics as spherical-cam designs with non-rotating circular cone followers or spherically-shaped followers. The flat-faced follower in Fig. 4 rotates about an axis that is the contact face rather than the center of the plane ring. The plane ring follower corresponds to the flat-faced follower in plane kinematics.

Closed-form guides. Besides having the follower contained as in Fig. 2, spherical-cam mechanisms can be designed so the cone roller on the follower is guided along the body of the input cam. For example, in Fig. 5, the cone roller moves along a groove that has been machined on the spherical inside surface of the input cam. However, this type of guide encounters difficulties unless the guide is carefully machined. The cone roller tends to seize.

Although cone rollers are recommended for better motion transfer between the input and output, there are some types of motion where their use is prohibited.

For instance, to obtain the motion diagram shown in Fig. 6, a cone roller would have to roll along a surface where any change in the concave section would be limited to the diameter of the roller. Otherwise there would be a point where the output motion would be interrupted. In contrast, the use of a knife-edge follower theoretically imposes no limit on the shape of the cam. However, one disadvantage with knife-edge followers is that they. unlike cone followers, slide and hence wear faster.

Manufacturing methods. Spherical cams are usually made by copying from a stencil. In turn, the cam-shaped tools can be copied from a stencil. Normally the cams are milled, but in special cases they are ground.

Three methods for manufacture are used to make the stencils:

- Electronically controlled point-by-point milling.
- Guided-motion machining.
- Manufacture by hand.

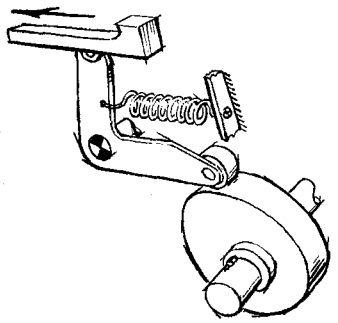
However, this last method is not recommended, because it isn't as accurate as the other two.

Tailored Cycloid Cams:

German Method

The cycloid cam is becoming the best all-around performer, but the problem is knowing how to fit it to specific machines requirements.

Nicholas P. Chironis



IT'S quite a trick to construct a cycloid curve to go through any point P within a cam diagram, *with a specific velocity at P* (Fig 1, opposite).

There is a growing demand for this type of modification because cam designers are turning more and more to the cycloid curve to meet most automatic machine requirements. They like the fact that a cycloid cam produces no abrupt change in acceleration and so induces the lowest degree of vibration, noise, shock, and wear. A cycloid cam also induces low side-thrust loads on a follower and requires small springs. However, the mathematical computations to tailor such cams become quite complex and the cycloid is all too often passed over for one of the more easily analyzed cams.

Recently, a well-known mechanism analyst at University of Bridgeport, Professor Preben W. Jensen, began a careful study through German cam design methods and came up with three graphical techniques for tailoring a cycloid cam, one of which solves the problem stated above. In an exclusive interview with PRODUCT ENGINEERING, Prof Jensen outlined the three common problems and the construction methods for solving them. He also provided the velocity and acceleration formulas for the cycloid, including the key relationship for keeping the maximum accelerations of the cam followers to a minimum. Specifically, the three types of tailoring are:

1) To have the cam follower start at point A, pass through P with a certain slope (velocity) and then proceed to point B—the entire motion to have cycloidal characteristics which includes zero acceleration slopes (smoothly starting velocities) at points A and B, Fig 1. (A cam diagram is actually a displacement record of the motion of a follower as it rises from point A to B during a specific rotation of the cam from line A to A'. Distance A-A' may be 180 deg or any other portion of the full rotation of the cam.)

2) To have the cam follower start with cycloid motion from point A, meet smoothly a constant velocity portion of the cam line (P_1-P_2 in Fig 2), and then continue on with cycloid motion to point B.

3) Given some other cam curve (curve AB in Fig 3), to return the follower to its starting point with cycloid motion (curve BMD).

Going through any point

This is the first of the modifications. The method of construction is:

Step 1. Draw a line DE with the given slope at P in Fig 1B.

Step 2. Divide AP into a number of equal parts, say 6—the larger the number of parts into which the line divided, the higher the degree of accuracy of the method. From the midpoint M of line AP, draw a line to D. This gives a distance C_1.

Step 3. Calculate radius R_1 from the relationship

$$R_1 = \frac{2C_1}{\pi}$$

(The derivation of the above equation is beyond the scope of this article.)

Step 4. Draw a quarter circle with R_1 as its radius and divide it into 3 equal parts. By dropping perpendiculars, obtain distance y_1 and y_2.

Step 5. Lay out distances y_1, y_2, and R_1, as shown in the diagram. The points so determined are points on the modified cycloid.

The other part of the displacement curve from P to B is determined in exactly the same way with the aid of the other small diagram in which R_2 is the radius.

The acceleration curve resulting from this displacement curve (deter-

mined by the method shown later) is continuous.

Going through constant velocity

In this second modification, constant velocity motion is required from P_1 to P_2. With the same method as described previously, AP_1 and P_2B are connected with a modified cycloid, as shown in Fig 2, and again an acceleration curve is obtained which is continuous.

Slowing down from given curve

Suppose that the first part of the cam, curve AB in Fig 3, must employ a different type of cam contour. How do you retract the follower smoothly to D using the cycloid curve so as to have continuous acceleration?

Solution: Connect B with D and draw the tangent to the curve given at B. Divide BD into equally spaced parts, with midpoint at M. Choose the line of maximum slope FME. This slope determines the maximum velocity during the return of the follower. The rest of the construction is carried out exactly as in the first case.

Velocity and acceleration equations

For a given rotation of the cam (distance θ in Fig 4) the equation for a tailored cycloid which gives the distance y that the follower will move is:

$$y = h\left[\frac{\theta_m}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta_m}{\beta}\right)\right] \quad (1)$$

where distance θ_m is computed from the equation

$$\theta_m = \theta + \frac{\beta}{2\pi}\left(\frac{\tan\delta}{\tan\gamma}\right)\sin\left(2\pi\frac{\theta}{\beta}\right)$$

and where

γ = direction of amplitude for the superimposed sine wave; ie, the angle of 'distortion' of the cycloid. For example, in Fig 4, when $\gamma = 90$ deg, then $\theta_m = \theta$ and you have a pure cycloid curve.

δ = angle of slope for the line connecting A and B

θ = portion of cam rotation, deg (or inches when measured on diagram)

β = time for total lift, deg (or inches)

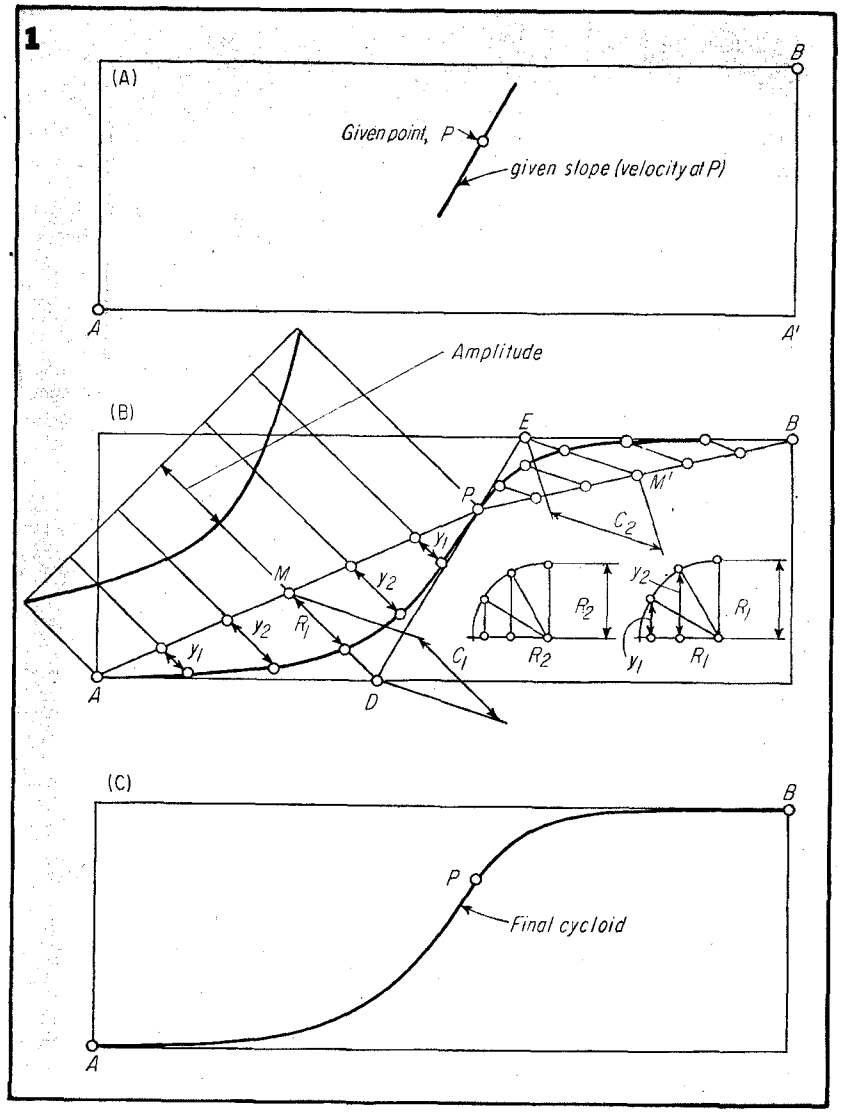
h = total follower movement, in.

$$T = \frac{60}{N}\left(\frac{\beta}{360}\right)$$

N = rpm of cam

Dimensions θ, θ_m and y are also in inches on the cam layout. Although Fig 4 shows P at the midpoint of B, the equations hold true for other cases

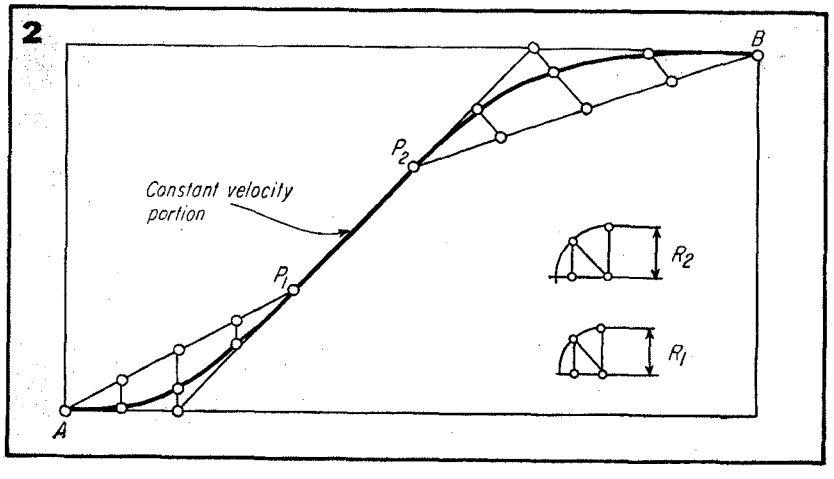
1



Technique for modifying a cycloid cam so that its follower speeds up smoothly to a specific velocity (slope at P) after extending a certain distance (to point P).

In the modification below, the cam follower is designed to move with a constant velocity during a portion of its stroke (line P_1P_2), as in cutting operations.

2



by proper modification of the value for T.

Velocity equation

$$V = \frac{h}{T}\left[\frac{1 - \cos\left(2\pi\frac{\theta}{\beta}\right)}{1 - \frac{\tan\delta}{\tan\gamma}\cos\left(2\pi\frac{\theta}{\beta}\right)}\right] \quad (2)$$

where V = follower velocity, in./sec

Acceleration equation

$$A = \frac{h}{T^2}\left[\frac{2\pi(1-K)\sin\left(2\pi\frac{\theta}{\beta}\right)}{\left[1 - K\cos\left(2\pi\frac{\theta}{\beta}\right)\right]^3}\right] \quad (3)$$

where $K = \tan\delta/\tan\gamma$

The stroke h and cam rotation β are usually fixed by the basic requirements of the problem. Therefore, the maximum acceleration, A, will depend upon K. There is one value of K which will give the lowest possible maximum value of acceleration. This optimum K value is

$$K_{\text{optimum}} = 1 - \tfrac{1}{2}\sqrt{3} = 0.134 \quad (4)$$

Comparison of cam curves

For the above optimum value of K the following minimum values of maximum acceleration are obtained:

For the best tailored cycloid cam

$$A_{max} = 5.89\frac{h}{T^2} \quad (5)$$

For a standard cycloid cam

$$A_{max} = 6.28\frac{h}{T^2} \quad (6)$$

For a parabolic cam

$$A_{max} = 4.00\frac{h}{T^2} \quad (7)$$

For a simple harmonic cam

$$A_{max} = 4.93\frac{h}{T^2} \quad (8)$$

Although the parabolic and simple-harmonic cams have lower acceleration maximums than the cycloids, their accelerations go through what is commonly referred to a "jerk," which is an abrupt change in acceleration (in these cases, from positive to negative values, see Fig 5).

Design example

A cam rotates with N = 200 rpm, the stroke of the follower is h = 2.0 in., and the corresponding cam shaft rotation is β = 60 deg. Determine the displacement, velocity, and acceleration curves for a best tailored cycloid having the lowest maximum acceleration.

Solution: In Fig 6 an arbitrary scale is chosen for the abscissa, namely that β = 60 deg has a length of 4 in. The stroke is laid out to scale (but establishing the stroke at a different scale would not change the procedure). Points A and B represent the start and end of the lift, respectively.

Angle δ is found from:

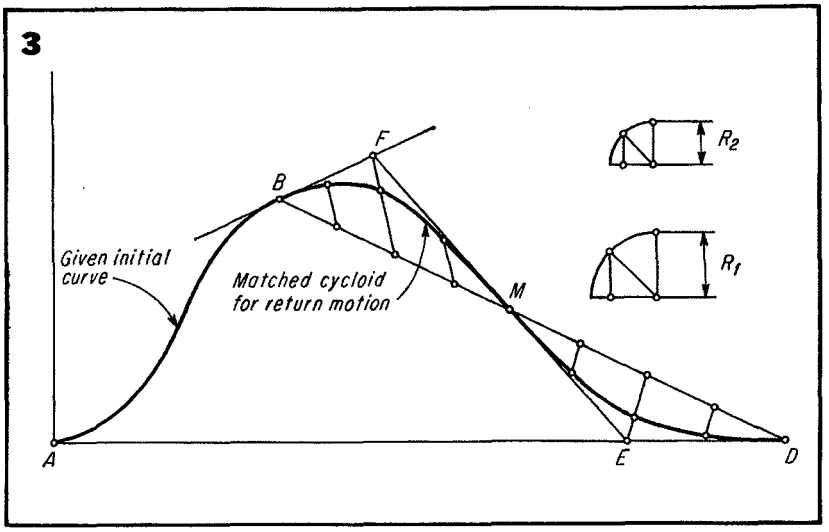
$$\tan\delta = \frac{h}{T} = \frac{2.00}{4.00} = 0.5$$

Because the lowest maximum acceleration is wanted, K = 0.134. Hence

$$\tan\gamma = \frac{\tan\delta}{K_{\text{optimum}}} = \frac{0.5}{0.134} = 3.73$$

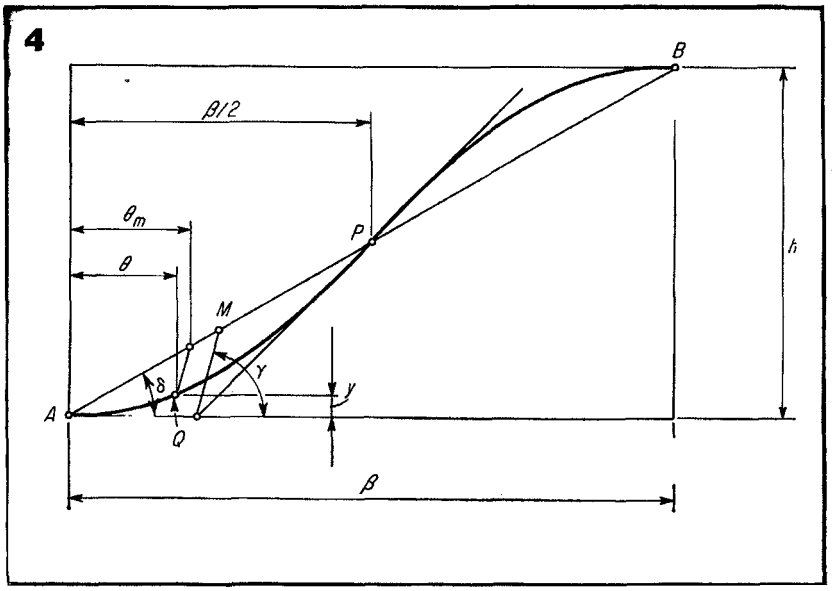
$$\tan\gamma = 75 \text{ deg}$$

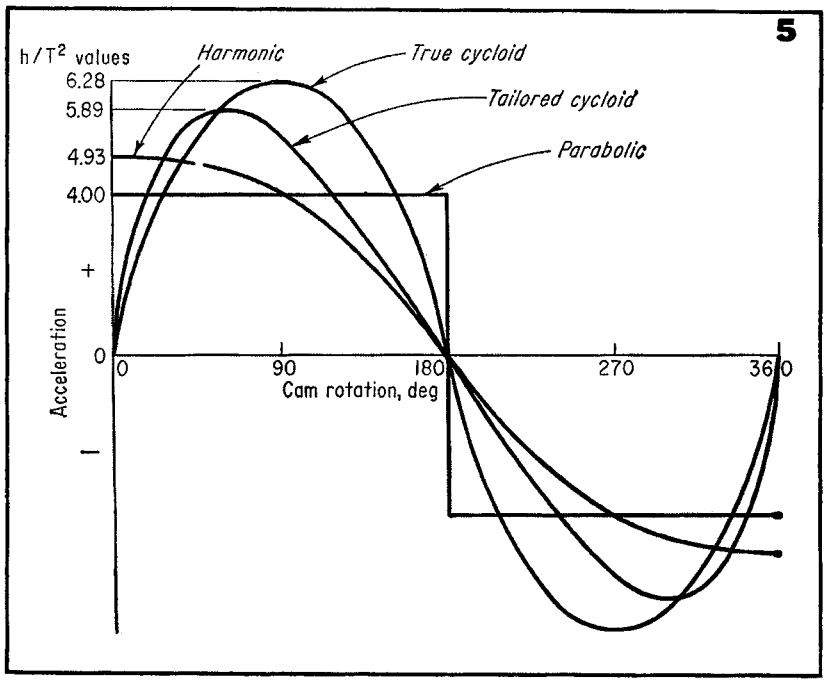
Referring again to Fig 6, P is the midpoint of AB, and AP is divided into 6 equal parts. Point D is situated so that line 3-D makes an angle of γ = 75 deg with the horizontal. This line indicates the direction of the amplitude of the sine wave which is superimposed on AP. The displace-

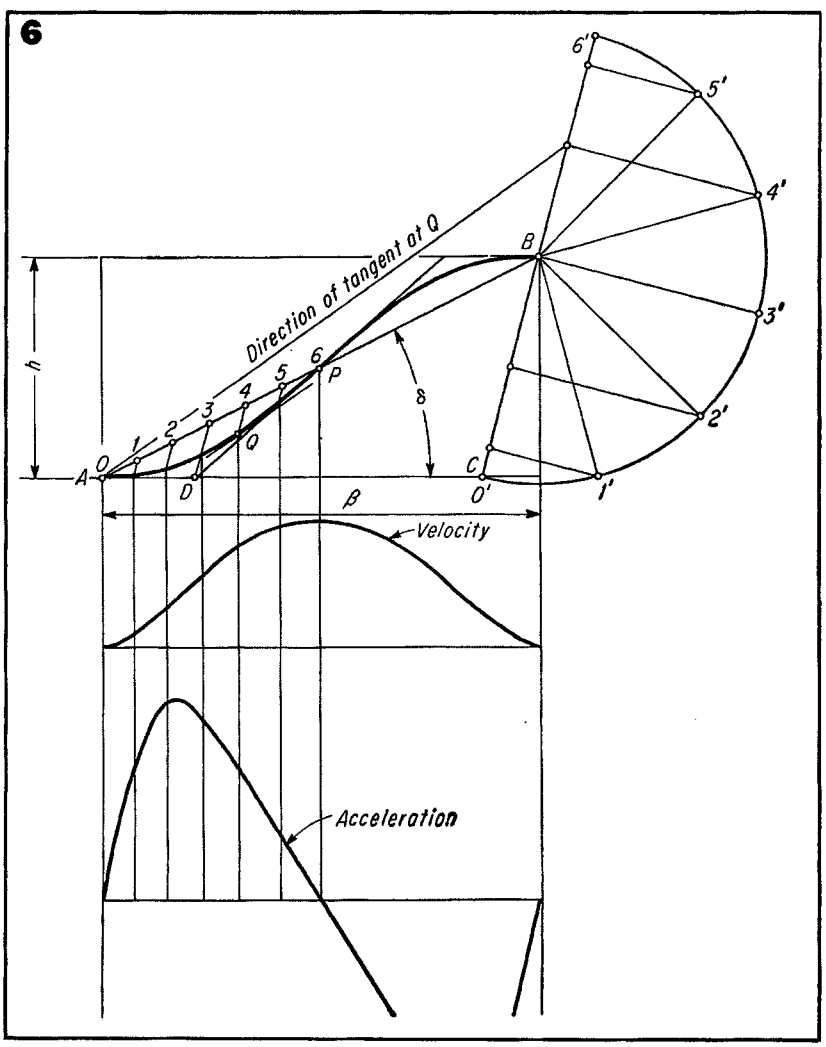


3. Graphical technique for slowing down the follower at the end of its work stroke (point B) and returning it to its initial position by means of the cycloid curve.

4. Basic cycloid curve (below) with factors which play a key role in finding its velocity and acceleration equations. When γ = 90 deg, the curve is a pure cycloid.







ment curve can now be drawn (the curve from 6 to B is congruent with the curve from A to 6).

The velocity at any point can be found from Eq 2, but can also be found graphically the following way:

Through B, draw line BC parallel to 3-D. With BC as radius, a half circle is drawn and divided into six equal parts. Now to find the velocity at point Q, draw a perpendicular from 4′ on BC and connect the point of intersection with A. This line is parallel to the tangent at Q. The procedure is repeated for the remaining points and the velocity curve is obtained.

To calculate the velocity at point Q by means of Eq 2, the value of θ for this point, is not $\theta = \beta/3$, but $\theta = 0.315\beta$ (see Fig 4). Hence

$$T = \frac{60}{N}\left[\frac{\beta}{360}\right]$$

$$= \frac{60(60)}{200(360)} = 0.05 \text{ sec}$$

and from Eq 2:

$$V = \frac{2.0}{0.05}\left[\frac{1 - \cos[2\pi(0.315)]}{1 - \left(\frac{0.5}{3.73}\right)\times \cos[2\pi(0.315)]}\right]$$

$$V = 56.3 \text{ in./sec}$$

The acceleration is found from Eq 3:

$$A = \frac{2(2\pi)(1 - 0.134)\sin 113.5°}{0.05^2(1 - 0.134\cos 113.5°)^3}$$

$$= 3320 \text{ in./sec}^2$$

The maximum acceleration, however, is more easily found from Eq 5:

$$A_{\max \text{ (optimum)}} = 5.89\frac{2}{0.05^2}$$

$$= 4710 \text{ in./sec}^2$$

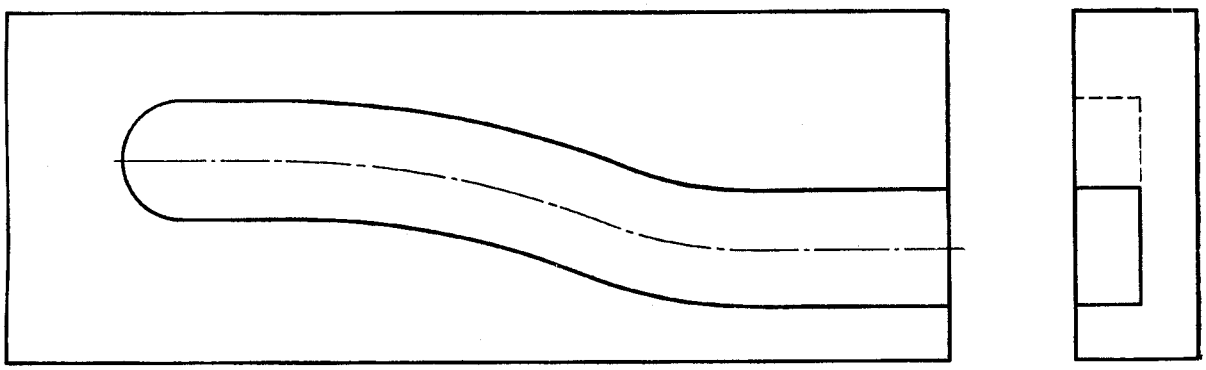
It is also of interest to notice that the maximum pressure angle of the modified cycloid is lower than that of the true cycloid. The angle for the modified curve can be measured from Fig 6 as approximately 41 deg. For a true cycloid it would be 45 deg.

5. Comparison of acceleration curves for four popular types of cam curves. The harmonic and parabolic curves have undesirable abrupt changes at 0 and 180 deg, respectively.

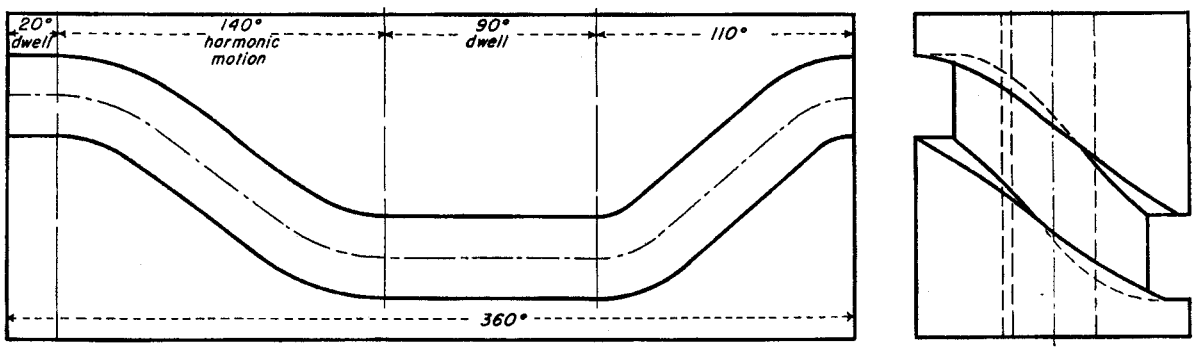
6. Construction details for finding the displacement, velocity and acceleration curve for a tailored cycloid cam.

Modifications and Uses for Basic Types of Cams

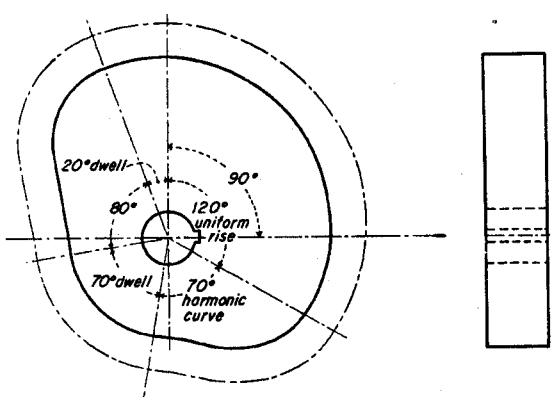
Edward Rahn



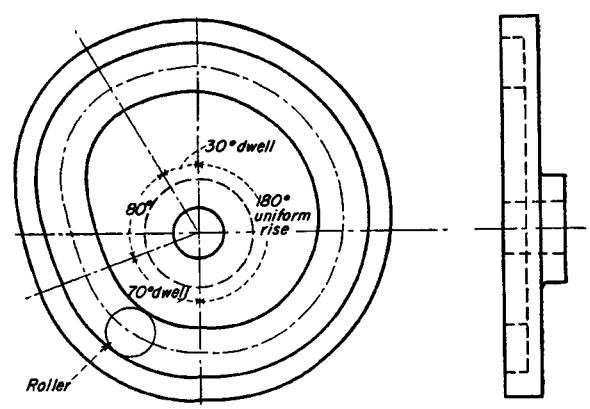
FLAT PLATE CAM—Essentially a displacement cam. With it, movement can be made from one point to another along any desired profile. Often used in place of taper attachments on lathes for form turning. Some have been built in sections up to 15 ft. long for turning the outside profile on gun barrels. Such cams can be made either on milling machines or profiling machines.



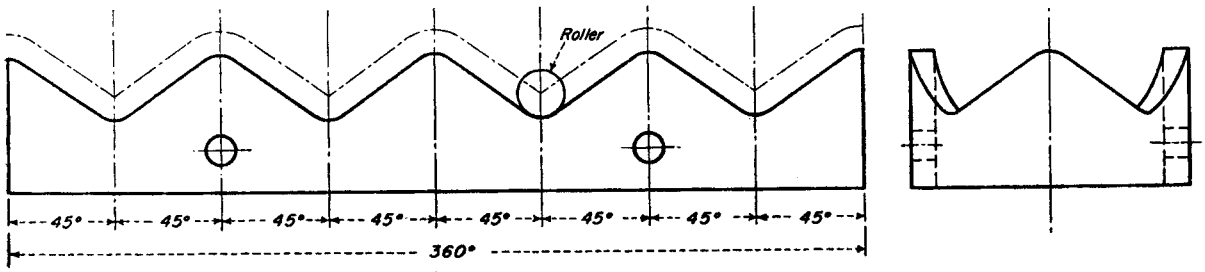
BARREL CAM—Sometimes called a cylindrical cam. The follower moves in a direction parallel to the cam axis and lever movement is reciprocating. As with other types of cam, the base curve can be varied to give any desired movement. Internal as well as external barrel cams are practical. A limitation: internal cams less than 11 in. in diam. are difficult to make on cam millers.



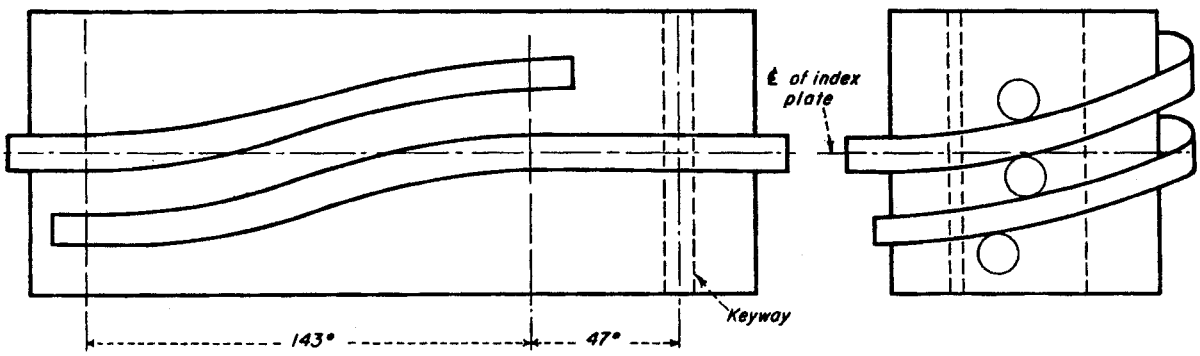
NON-UNIFORM FACE CAM—Sometimes called a disk cam. Follower can be either a roller, hexagon or pointed bar. Profile can be derived from a straight line, modified straight line, harmonic, parabolic or non-uniform base curve. Generally, the shock imposed by a cam designed on a straight line base curve is undesirable. Follower usually is weight loaded, although spring, hydraulic or pneumatic loading is satisfactory.



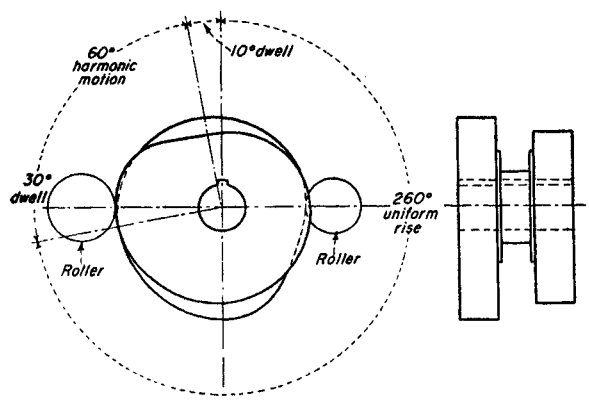
BOX CAM—Gives positive movement in two directions. A profile can be based on any desired base curve, as with face cams, but a cam miller is needed to cut it; whereas with face cams, a band saw and disk grinder could conceivably be used. No spring, pneumatic or hydraulic loading is needed for the followers. This type cam requires more material than for a face cam, but is no more expensive to mill.



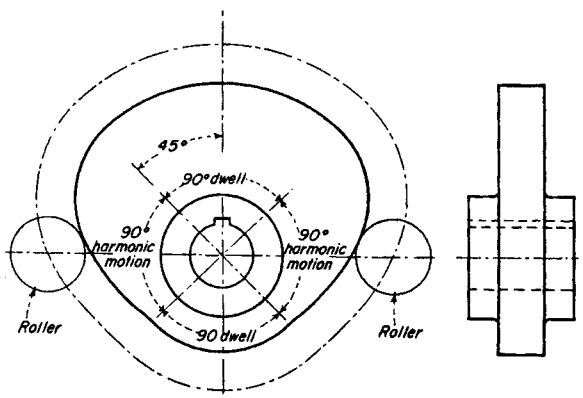
SIDE CAM—Essentially a barrel cam having only one side. Can be designed for any type of motion, depending on requirements and speed of operation. Spring or weight loaded followers of either the pointed or roller type can be used. Either vertical or horizontal mounting is permissible. Cutting of the profile is usually done on a shaper or a cam miller equipped with a small diameter cutter, although large cams 24 in. in diameter are made with 7-in. cutters.



INDEX CAM—Within limits, such cams can be designed for any desired acceleration, deceleration and dwell period. A relatively short period for acceleration can be alloted on high speed cams such as those used on zipper-making equipment on which indexing occurs 1,200 to 1,500 times per minute. Cams of this sort can also be designed with four or more index stations.



DOUBLE FACE CAM—Similar to single face cam except that it provides positive straight line movement in two directions. The supporting fork for the rollers can be mounted separately or between the faces. If the fork fulcrum is extended beyond the pivot point, the cam can be used for oscillatory movement. With this cam, the return stroke on a machine can be run faster than the feed stroke. Cost is more than that for a box cam



SINGLE-FACE CAM WITH TWO FOLLOWERS—Similar in action to a box or double face cam except flexibility is less than that for the latter type. Cam action for feed and return motions must be the same to prevent looseness of cam action. Used in place of box cams or double face cams to conserve space, and instead of single face cams to provide more positive movement for the roller followers.